The t Test for Two Related Samples

Types of Two-Sample t Tests

- Two-sample t tests can be either
  - *Repeated measures* – a single sample of individuals is measured more than once on the same dependent variable
  - *Matched subjects* – each individual in one sample is matched with an individual in another sample. Matching is done so that the two individuals are nearly equivalent with respect to a specific variable that the researcher wants to control

Related Samples

- The t test for related samples is similar to the single sample t test
- Calculate the difference score for each individual
  - $D = X_2 - X_1$
  - Use the difference scores as the variable in the single sample t test
Two-Sample t-Tests

\[ t = \frac{M_D - \mu_D}{s_{M_D}} \]
\[ s_{M_D} = \sqrt{\frac{s^2}{n}} \]

Example

A researcher measures how many of 20 nouns people can remember

Then people are taught a mnemonic technique

Researcher measures how many of 20 nouns the same people can remember using the mnemonic

Does the mnemonic change memory?

<table>
<thead>
<tr>
<th>Person</th>
<th>No Mnemonic</th>
<th>With Mnemonic</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
<td>7</td>
<td>-1</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>10</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>F</td>
<td>7</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>9</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>6</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>I</td>
<td>9</td>
<td>8</td>
<td>-1</td>
</tr>
<tr>
<td>J</td>
<td>11</td>
<td>15</td>
<td>4</td>
</tr>
</tbody>
</table>

Step 1

Write the hypotheses:

\( H_0: \mu_D = 0 \)

This states that the mnemonic had no effect – the two scores per individual are equal

\( H_1: \mu_D \neq 0 \)

This states that the mnemonic had an effect – one of the two scores is larger, on average, than the other

Specify \( \alpha \)

\( \alpha = .05 \)
Step 2

- Find the critical region(s)
  - This is a two-tailed test as the direction of the effect is not specified – only that the mnemonic should change memory (make it better or make it worse)
  - df = n – 1
    - df = 10 – 1 = 9
  - Consult a table of critical t values
    - The critical t = 2.262

Step 2

- If the calculated t is larger than 2.262 or smaller than -2.262, then it is in the tail cut off by the critical t and we should reject $H_0$

Step 3

<table>
<thead>
<tr>
<th>Person</th>
<th>No. Mnemonic</th>
<th>With Mnemonic</th>
<th>Difference, D</th>
<th>$dD^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>0.96</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>9</td>
<td>2</td>
<td>0.36</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
<td>7</td>
<td>-1</td>
<td>5.76</td>
</tr>
<tr>
<td>D</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td>2.56</td>
</tr>
<tr>
<td>E</td>
<td>10</td>
<td>13</td>
<td>3</td>
<td>2.56</td>
</tr>
<tr>
<td>F</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>0.16</td>
</tr>
<tr>
<td>G</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>1.96</td>
</tr>
<tr>
<td>H</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td>2.56</td>
</tr>
<tr>
<td>I</td>
<td>9</td>
<td>8</td>
<td>-1</td>
<td>5.76</td>
</tr>
<tr>
<td>J</td>
<td>11</td>
<td>15</td>
<td>4</td>
<td>6.76</td>
</tr>
</tbody>
</table>

$D = 14, \bar{X} = 1.4, SS = 30.4$

$t = \frac{M_{D} - \bar{D}}{s_{D}} = \frac{1.4 - 0}{0.5812} = 2.409$

$s_{D} = \sqrt{\frac{\sum D^2}{n^{2} - 1}} = \sqrt{\frac{3.3778}{10 - 1}} = 0.5812$

$y^2 = \frac{SS}{n - 1} = \frac{30.4}{9} = 3.3778$
Step 4

- Make a decision
  - The calculated $t$ (2.409) is in the critical region; it is in the tail cut off by the critical $t$ (2.262)
  - Therefore, reject $H_0$
    - It is likely the case that the mnemonic had an effect on memory
- SPSS reports $p = .039$.
  - Reject $H_0$ because $p \leq \alpha$

Effect Size

Cohen's $d = \frac{\text{population mean difference}}{\text{standard deviation}} = \frac{\mu_2 - \mu_0}{\sigma_0}$

Estimated $d = \frac{\text{sample mean difference}}{\text{sample standard deviation}} = \frac{M_2 - M_0}{s}$

Estimated $d = \frac{M_2 - M_0}{s} = \frac{1.4}{\sqrt{3.3778}} = 0.76$

Effect Size

$$r^2 = \frac{t^2}{t^2 + df}$$

$$r^2 = \frac{2.409^2}{2.409^2 + 9} = 0.392$$
Confidence Interval

© $\mu_D = \bar{M}_D \pm t \cdot s_M$

© $95\% \text{ CI} = 1.4 \pm 0.5812$  
$95\% \text{ CI} = 0.805$ to $2.715$

© Reject $H_0$ because the confidence interval does not contain 0.

© What happens to the width of the CI as the confidence level increases (e.g. 95 to 99% CI)?

© What happens to the width of the CI as the sample size increases?

Uses of Repeated Measures Designs

© Repeated measures designs

© tend to use fewer participants than an independent samples design

© are good for studying effects that change across time

© reduce individual differences

© variance is typically smaller in a repeated measures design than in an independent samples design

Time Related Factors / Order Effects

© Because each individual is measured at two points in time in a repeated measures design, time of measurement can become confounded with the independent variable

© Confound – a variable that should be constant but varies simultaneously with the independent variable

© Confounds limit your ability to make statements of causality
Time Related Factors / Order Effects

- *Counterbalancing* is a set of techniques that attempt to keep these time related factors from becoming confounded with the independent variable.
  - Half get condition 1 followed by condition 2
  - Half get condition 2 followed by condition 1

Assumption of Repeated Measures t

- The observations *within* each treatment must be independent.
- The population distribution of difference scores (D) must be normal.
  - This is not a concern when the same size is greater than 30.