Aeroelastic Design Optimization using a Multifidelity Quasi-Newton Method

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The traditional aircraft design process relies upon low-fidelity models for expedience and resource savings. However, the reduced accuracy and reliability of low-fidelity tools often lead to the discovery of design defects or inadequacies late in the design process. These deficiencies result either in costly changes or the acceptance of a configuration that does not meet expectations. Multifidelity methods attempt to blend the increased accuracy and reliability of high-fidelity models with the reduced cost of low-fidelity models. A new multifidelity algorithm has been proposed, combining elements from typical Trust Region Model Management and classical quasi-Newton methods. In this paper, the algorithm is compared to the single-fidelity quasi-Newton method for complex aeroelastic simulations. The vehicle design problem includes variables for planform shape, structural sizing, and cruise condition with constraints on trim and structural stresses. Considering the objective function reduction versus computational expenditure, the multifidelity process performs better in three of four cases in early iterations. However, the enforcement of a contracting trust region slows the multifidelity progress. Even so, leveraging the approximate inverse Hessian, the optimization can be seamlessly continued using high-fidelity data alone. Ultimately, the proposed new algorithm produced better designs in all four cases. Investigating the return on investment confirms that the multifidelity advantage is greatest in early iterations, and managing the transition to high-fidelity optimization is critical.

Nomenclature

\begin{align*}
AR & \quad \text{Aspect ratio} \\
C & \quad \text{Vehicle specific fuel consumption} \\
C_D & \quad \text{Vehicle drag coefficient} \\
C_L & \quad \text{Vehicle lift coefficient} \\
c & \quad \text{Element-wise stress constraint} \\
f, f_h, f_l & \quad \text{Range optimization objective function (high- and low-fidelity)} \\
g & \quad \text{Optimization inequality constraint} \\
h & \quad \text{Optimization equality constraint} \\
KS & \quad \text{Kreisselmeier-Steinhauser stress constraint aggregation function} \\
m_f & \quad \text{Vehicle final mass} \\
q & \quad \text{Dynamic pressure} \\
R & \quad \text{Vehicle range} \\
S & \quad \text{Vehicle planform area} \\
t & \quad \text{Material gauge} \\
V & \quad \text{Vehicle velocity} \\
W_f & \quad \text{Vehicle final (empty) weight} \\
W_i & \quad \text{Vehicle initial (gross) weight} \\
w & \quad \text{Penalty function weight} \\
x & \quad \text{Design variable vector} \\
x_c & \quad \text{Current design (center of trust region)}
\end{align*}

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I. Introduction

A model’s level of fidelity may be defined as its accuracy in faithfully reproducing a quantity or behavior of interest of a real system. As statistician George Box once noted, “All models are wrong, but some are useful.” Fidelity levels fall on a multi-dimensional spectrum. Variations may include, for example, changes in physics modeled, inclusion or coupling of different disciplines, variation in geometric detail, faithfulness of boundary and loading conditions, or reduction of numerical error in a solution process. What constitutes a change in fidelity depends on the intended use and the user. For some, introduction of new disciplines fundamentally changes the behavior of the designed system, whereas subsystem-level predictions may be agnostic. For some, changes in domain discretization and numerical error level have no impact on decision making, while for others such a change might result in a fundamentally different decision process. The traditional aircraft design process relies upon low-fidelity models for expedience and resource savings. However, the reduced accuracy and reliability of low-fidelity tools often lead to the discovery of design defects or inadequacies late in the design process. These deficiencies result either in costly changes or the acceptance of a configuration that does not meet expectations.

Examples of applying multifidelity concepts to gradient-based optimization in the literature are based on the Trust Region Model Management (TRMM) approach presented by Lewis and Alexandrov et al. for unconstrained optimization. Design constraints are explicitly considered by Rodríguez, Renaud, and Watson and Alexandrov et al. Also extended the TRMM concepts to a general Approximation and Model Management Optimization framework, demonstrating it for augmented Lagrangian optimization, the multilevel algorithms for large-scale constrained optimization (MAESTRO) framework, and a trust-region Sequential Quadratic Programming (SQP) method. TRMM is built on a hierarchy of data in which one fidelity is trusted more than another. Two underlying assumptions are that the low-fidelity model is sufficiently accurate to provide useful trends, and that the cost of the low-fidelity evaluation is (much) less than that of the high-fidelity evaluation. Thus, a method is sought to intensely search the low-fidelity model to reduce the number of high-fidelity evaluations required.

While allowing any optimizer to be used as a black box, there are several weaknesses of TRMM. First, aside from the sizing of the trust region and the matching of the high-fidelity objective and gradient at the center of the new subproblem, no information is retained between sub-optimizations. Ideally, once the expensive truth function is evaluated, its value and gradient would be used to guide further progress by the optimizer. Second, the efficacy of the approach is highly dependent on the quality of the approximate model. Building accurate, high-dimensional approximate models over large regions of the design space is difficult, and is a vast research area in and of itself. Third, while TRMM is provably convergent to an optimum of the high-fidelity problem, the rate of convergence in the neighborhood of the optimum is not guaranteed. A study on rates of convergence is provided by Eldred, Giunta, and Collis. Finally, each iteration of TRMM may require many low-fidelity function evaluations. While the lower-fidelity prediction is cheaper than that of the high-fidelity, its cost may not be negligible. To the contrary, in order to obtain better approximations of the truth function, a higher-fidelity approximation may be required, making each evaluation more costly.

To address these weaknesses, Bryson and Rumpfkeil proposed a new gradient-based, multifidelity opti-
mization method blending elements of quasi-Newton methods and TRMM. A key distinction of this approach is that it maintains an estimate of the approximate inverse Hessian, which represents the curvature of the objective function. Whereas TRMM performs a full sub-optimization at every iteration, the multifidelity quasi-Newton method finds a suitable descent direction and performs a line search using the approximate model. This paradigm shift provides three key advantages. First, the valuable, previously-evaluated high-fidelity data are leveraged to find a search direction rather than being discarded. Second, the burden on surrogate models is reduced from being accurate in a hyper-volume bounded by the trust region to being accurate along a line in a forward-looking direction. Third, the multifidelity optimization may be seamlessly transitioned to a fully high-fidelity process at any time to fine tune the design or if the disagreement between the fidelity levels inhibits progress.

An alternative to gradient-based, multifidelity optimization is global optimization on multifidelity surrogates, frequently termed Efficient Global Optimization (EGO). In this approach, a surrogate model (typically kriging) is constructed from multiple data sources, and searched using a global optimizer. Candidate points are selected to be evaluated with the high-fidelity function, and this truth data is included in a rebuilding of the surrogate model. There are many selection criteria, with the goal of balancing exploration and exploitation. A popular criterion is the expected improvement, which combines the anticipated response value with an estimate of uncertainty in the surrogate. However, these global search methods require many function evaluations, and are sensitive to the initial sampling plan and infill selection criterion.

This paper considers the optimal aero-structural shape and sizing design of a lambda wing vehicle to maximize its range. The aeroelastic vehicle deformation is included to provide a more complete picture of performance, and multifidelity methods are used to provide high-fidelity predictions accelerated by leveraging low-fidelity data. While the multidisciplinary couplings here are limited to aeroelasticity, the design methods are extensible to include other disciplines such as propulsion as well as stability and control. The optimization serves as a multidisciplinary demonstration of the multifidelity quasi-Newton method proposed by Bryson and Rumpfkeil, and the optimizer performance is compared to that of using the high-fidelity model alone.

II. Methodology

A. Optimization Problem Definition

As a demonstration problem, a preliminary design-level multifidelity, multidisciplinary optimization is formulated using static aeroelastic analyses. The configuration under consideration (depicted in Figure 1) is based on the work of Alyanak, Pendleton, and Allison. The planform area is held constant at 230 m² for the full vehicle, and it is assumed that the half-span gross take-off mass is constant at 23,000 kg. This assumption is enforced between different configurations by varying the fuel weight. Range optimization is considered here at Mach 1.2, with a dynamic pressure of 19,486.6 Pa. The multidisciplinary design variables include the planform geometry, structural gauges, and cruise angle of attack. The higher-fidelity analysis consists of body-fitted Euler CFD coupled with linear structural mode shapes; the lower-fidelity analysis implements panel aerodynamics tightly coupled with full, linear structural FEA.

Figure 1. Outer mold line and internal layout of the baseline lambda wing vehicle.
The aircraft range, $R$, may be calculated using the Breguet range equation,

$$R(x) = \frac{V}{C} \frac{C_L(x)}{C_D(x)} \ln \frac{W_i(x)}{W_f},$$

(1)

where $V$ and $C$ are the velocity and specific fuel consumption, respectively, $\frac{C_L}{C_D}$ is the lift-to-drag ratio, and $\frac{W_i}{W_f}$ is the ratio of the initial weight to the final weight. It is assumed that the lift-to-drag ratio is computed near the middle of cruise with a 50% fuel load. Because the vehicle must trim for steady, level flight, the lift may be replaced by the weight at this design condition, $W_{0.5} = \frac{1}{2}(W_i + W_f)$. Constant specific fuel consumption is also assumed in the derivation of the Breguet range equation, further simplifying the problem. These assumptions yield the relationship

$$\frac{CR(x)}{V} = \frac{1}{2}(W_i(x) + W_f) \ln \frac{W_i(x)}{W_f}.$$ 

(2)

To form the problem as a standard minimization, the reciprocal is taken, making the objective function linear in drag and avoiding numerical difficulties as drag approaches zero during optimization. Thus, the objective function becomes

$$f(x) = \frac{qSC_D(x)}{\frac{1}{2}(W_i(x) + W_f) \ln \frac{W_i(x)}{W_f}}.$$ 

(3)

The aerodynamic performance is driven by the vehicle outer mold line coupled with the aeroelastic deformations, while the weight performance is driven by the structures discipline. The vehicle weight at the end of cruise, $W_f$, equals the weight of the structure, payload, and subsystems (estimated by textbook methods), of which the latter two are again taken from Alyanak and Pendleton.\textsuperscript{17}

The optimal designs must satisfy several constraints to be valid. First, at the most basic level, the lift must equal the aircraft weight at the nominal design condition,

$$h_1(x) = \frac{1}{2}(W_i(x) + W_f) \frac{qSC_L(x)}{qSC_D(x)} - 1 = 0.$$ 

(4)

The lift appears in the denominator so that the constraint may also be considered as an inequality constraint where excess lift is not penalized, with the drag-minimization aspect of the problem driving the trim to its active boundary. Both constraint variations are explored in Section III. As a simplification to the geometry and aeroelastic analysis, the vehicle is defined without control surfaces, which would be used to trim the moments, and thus the moments are not constrained.

Second, the structure must bear the applied loads without failing. The design load case is a 9-g pull-up maneuver, once again at the 50% fuel condition. On an element-by-element basis, the stress constraints may be defined based on the von Mises criterion, $\sigma_{VM,i}$, and failure stresses, $\sigma_F$, (taken to be 490 MPa for high-strength aluminum 7075-T6, a common alloy in wing upper skins\textsuperscript{20}),

$$c_i(x) = \frac{\sigma_{VM,i}(x)}{\sigma_F,i} - 1 \leq 0.$$ 

(5)

It should be noted here that the generation of analysis meshes utilizes a remeshing approach rather than morphing the meshes. This approach is taken for two reasons. First, morphing the meshes of individual disciplines and fidelity levels would lead to inconsistencies in the underlying geometry, which is used to perform interdisciplinary data transfers. Second, large shape variations are both expected and desired, which can result in mesh quality issues with morphing.

Considering the stress constraints on an element-by-element basis introduces a large number of constraints, and by using the remeshing approach, the number of constraints would be inconsistent over the course of the optimization. To address this difficulty, the Kreisselmeier-Steinhauser (KS) aggregation technique\textsuperscript{21,22} combines stress constraints over patches of structural elements. For this problem, the stress constraints are grouped into outboard skins, spars, and ribs. Following the formulation of Poon and Martins,\textsuperscript{21} the aggregated inequality constraints are defined as

$$g_j(x) = KS_j(x) = c_{max}(x) + \frac{1}{50} \ln \sum_{i=1}^{N_{elem}} e^{50(c_i(x) - c_{max}(x))} \leq 0, \quad j = 1, 2, 3.$$ 

(6)
While Poon and Martins provide an adaptive scheme for the weighting factor, here it maintains a value of 50 based on their recommendation of starting value. In the absence of aeroelastic divergence analysis, constraints on the wing deflection are also prudent. Limiting the magnitude of the tip displacement, the constraint is expressed as

\[ g_4(x) = \frac{\|\delta_{tip}(x)\|_2}{1.5 \text{ m}} - 1 \leq 0. \]  

(7)

The limit value of 1.5 m is selected based on preliminary analyses at the design space bounds.

The optimizer must also be constrained from choosing physically unreasonable designs, especially during large initial steps. First, the lift should be constrained to be positive,

\[ g_5(x) = -10C_L(x) \leq 0. \]

(8)

This constraint is redundant when the trim constraint \( h_1 \) is enforced as an equality condition, but becomes necessary when the constraint is relaxed to inequality. The factor of 10 is used to scale the response to order 1. The fuel mass, \( m_f \), must also be positive, i.e., the final mass must be less than the initial mass. This constraint may be expressed as

\[ g_6(x) = -\frac{m_f(x)}{5000 \text{ kg}} \leq 0. \]

(9)

The scaling of 5000 kg is chosen again to make the constraint on the order of 1.

The design variables and bounds are listed in Table 1. To reduce the size of the design problem for this initial demonstration, the skin gauges are symmetric on upper and lower surfaces, and are constant on three patches demarcated by the leading edge breaks. The material gauges of the internal structure are also separated into patches. The spar gauges are constant over two separate patches—the outboard wing and the centerbody—as are the ribs. The centerbody bulkheads also have a uniform gauge. The shape variables include aspect ratio, outboard sweep angle, and outboard taper ratio. Note that the aspect ratio variable seems very limited; however, because the entire lifting body is included in the reference area, this choice produces spans ranging from 10 to 30 m. The centerbody is held constant for packaging considerations, though the sweep angle of the midboard leading edge varies slightly as the chord of the outboard wing varies. From a flight dynamics perspective, the angle of attack is a design variable free to satisfy the trim condition.

The multifidelity optimizer currently implemented\(^\text{12}\) is an unconstrained optimization algorithm. (The multifidelity approach taken may be extended to constrained algorithms, which is a task for future work.) Thus, to introduce the constraints into the problem, a quadratic penalty function is used. In summary,

\[
\begin{align*}
\text{minimize} \quad & \phi(x) = f(x) + w_h h_1^2(x) + \sum_{i=1}^{6} w_{g_i} \max (g_i(x), 0)^2 \\
\text{with respect to} \quad & x = [\alpha, AR, \Lambda_{out}, t_{skin}, t_{spar}, t_{rib}] \\
\text{subject to} \quad & x_i^L \leq x_i \leq x_i^U.
\end{align*}
\]

(10)

The penalty weights were selected based on the relative magnitudes of the responses at the baseline configuration. With the high-fidelity objective function being on the order of 1, the aggregated stress constraints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of Attack ( \alpha )</td>
<td>-3</td>
<td>3</td>
<td>degree</td>
</tr>
<tr>
<td>Aspect Ratio ( AR )</td>
<td>2</td>
<td>4</td>
<td>—</td>
</tr>
<tr>
<td>Sweep ( \Lambda )</td>
<td>35</td>
<td>60</td>
<td>degree</td>
</tr>
<tr>
<td>Taper Ratio ( \lambda )</td>
<td>0.2</td>
<td>0.5</td>
<td>—</td>
</tr>
<tr>
<td>Skin Gauge ( t_{skin} )</td>
<td>0.005</td>
<td>0.05</td>
<td>m</td>
</tr>
<tr>
<td>Spar Gauge ( t_{spar} )</td>
<td>0.005</td>
<td>0.05</td>
<td>m</td>
</tr>
<tr>
<td>Rib Gauge ( t_{rib} )</td>
<td>0.005</td>
<td>0.05</td>
<td>m</td>
</tr>
</tbody>
</table>
and tip displacement constraints are weighted by a factor of 10. The constraints on the positivity of the lift coefficient and fuel mass are weighted by a factor of 100 to steer the optimizer strongly away from infeasible regions. When the lift constraint is enforced as equality, the weighting is a factor of 10 so that its effect is not so strong as to inhibit the optimizer, whereas the weighting is a factor of 100 when the constraint is considered an inequality to provide strong enforcement.

The behavior of the reciprocal range function (Eqn. 3) is plotted as a function of vehicle empty mass in Figure 2 for several values of drag coefficient. As expected, the objective is improved by reducing the empty mass and the drag coefficient. When the empty mass approaches the fixed gross weight of 23,000 kg, the objective grows unbounded as the available fuel mass vanishes. The relative benefits of reducing the mass versus the drag depends on the location in the design space. If the design is near its gross mass, the optimizer will favor reducing empty mass rather than drag if the trends are in opposition. On the other hand, if the empty mass is relatively low, the optimizer may see more benefit in reducing drag. These trends help to provide insight into the optimization results.

![Figure 2. Behavior of objective function for different drag coefficient values.](image)

**B. Multifidelity, Multidisciplinary Analysis and Optimization**

The responses for the optimization problem defined above are calculated from the multifidelity, multidisciplinary framework presented by Bryson and Rumpfkeil\(^{23}\) and summarized here for completeness. For additional details, the reader is referred to the original source. An overview of the analysis process is depicted in Figure 3. The process begins with the selection of design variables via an optimizer, parametric study, or other means. The model configuration and geometry generator, Computational Aircraft Prototype Syntheses (CAPS),\(^{24}\) interprets the parameters into the required aerodynamic and structural analysis models. These models feed the subsequent disciplinary analyses.

The first analysis is the evaluation of the structural weights of the as-designed model using the Automated STRuctural Optimization System (ASTROS).\(^{25, 26}\) It is reemphasized that the gross takeoff weight is defined to be constant across all configurations. The structural weights combined with fixed estimates of subsystem weights yield the empty weight used in the range calculation. The margin between takeoff and empty weights indicates the available fuel weight, of which 50% is used for the design maneuver and trim weight. The structural analysis model is updated to reflect this design weight before running the maneuver loads analysis in ASTROS. This analysis determines the structural stresses and strains by trimming (angle of attack only) the vehicle to achieve a prescribed load factor. Since the assumption is made that the upper and lower skins have the same material gauges, a 9-g pull-up maneuver is sufficient for the analysis.

The same structural analysis model also feeds the subsequent multifidelity static aeroelastic evaluation. All fidelities implemented are required to produce lift, drag, and tip displacement, which will be used in the objective and constraint evaluations. The lower-fidelity tool is ASTROS, which combines linear FEA with
Figure 3. Logical connections and fidelities in multifidelity, multidisciplinary analysis.

Panel aerodynamics. It generates structural deformations, lift, and induced drag. The higher-fidelity tool is Fully Unstructured Navier-Stokes 3D (FUN3D)\textsuperscript{27,28} in Euler mode with modal structures calculated again by ASTROS. The loads transfer and mesh deformation are handled internally by FUN3D.

The aeroelastic optimization requires coupled gradients of the response functions. ASTROS provides sensitivities of responses with respect to the material gauges, and FUN3D provides rigid aerodynamic sensitivities with respect to shape parameters. However the efficient calculation of coupled gradients is a topic of ongoing research.\textsuperscript{29,30} Here, finite differences are used as a demonstration. More efficient and accurate methods can be substituted as the tools become available and are integrated into CAPS.

1. Multifidelity Optimization Algorithm

The multifidelity optimization algorithm used for this design problem is the unified, multifidelity quasi-Newton approach of Bryson and Rumpfkeil,\textsuperscript{32} which combines elements of TRMM\textsuperscript{2,3} with the limited-memory, bound-constrained BFGS quasi-Newton (L-BFGS-B) method.\textsuperscript{31,32} The multifidelity quasi-Newton approach is compared graphically to TRMM in Figure 4. While the new approach is currently implemented in L-BFGS-B, it may be applied to any optimizer using a line search for globalization.

With each iteration, the optimizer uses the current gradient and approximate inverse Hessian to determine a search direction toward the expected optimal point, which minimizes the internal quadratic subproblem subject to trust region bounds. Using this quasi-Newton step, surrogate corrections are built forcing the low-fidelity model to match the high-fidelity objective and gradient at the current design and a previously computed design nearest the expected point. (In the first iteration, the actual expected point is computed using the high-fidelity model for startup purposes.) For the results presented here, the hybrid bridge polynomial corrections of Bryson and Rumpfkeil\textsuperscript{33} are implemented, though any method satisfying zeroth- and first-order consistency at the current expansion point (e.g., kriging) may be used. A multifidelity line search is conducted using the corrected low-fidelity model. The resulting point is accepted or rejected based on the reduction (or increase) of the true objective function, and the trust region size is updated using typical heuristics. In either case, the new high-fidelity data is used to update the approximate inverse Hessian, and the procedure is repeated until the termination criteria are reached. The specific heuristic for the trust region size in iteration $k + 1$, $\Delta^{(k+1)}$, is based on the ratio of actual improvement to expected improvement: \textsuperscript{2}
Figure 4. Comparison of TRMM to proposed unified approach. Dots with solid lines represent optimization path. Bold dashed lines show the side constraints, and light dashed lines are objective function contours. Gray boxes/lines display the approximate model domain for sub-optimization problems (TRMM) or line searches (Unified). Light solid lines illustrate trust region size for each iteration. Note that the trust region encompasses the entire domain starting with the second iteration for the unified approach.

\[ \rho^{(k)} = \frac{f(x_c^{(k)}) - f(x^*_c)}{\hat{f}(x_c^{(k)}) - \hat{f}(x^*_c)}. \]  

\[ \Delta^{(k+1)} = \begin{cases} 
0.5\Delta^{(k)} & \text{if } \rho^{(k)} \leq 0.25 \\
\Delta^{(k)} & \text{if } 0.25 < \rho^{(k)} \leq 0.75 \\
\gamma\Delta^{(k)} & \text{if } 0.75 < \rho^{(k)} \leq 1.25 \\
\Delta^{(k)} & \text{if } 1.25 < \rho^{(k)} \leq 1.75 \\
0.5\Delta^{(k)} & \text{if } 1.75 < \rho^{(k)}. 
\end{cases} \]  

\[ \gamma = \begin{cases} 
2 & \text{if } \|x^*_c - x_c^{(k)}\|_\infty = \Delta^{(k)} \\
1 & \text{if } \|x^*_c - x_c^{(k)}\|_\infty < \Delta^{(k)}. 
\end{cases} \]

The decision to contract the trust region when the actual improvement was much greater than anticipated is conservative in that it emphasizes the construction of accurate surrogates over luck.

Bryson and Rumpfkeil\textsuperscript{12} compared the performance of the new multifidelity optimizer, TRMM, and monofidelity BFGS for a set of five different analytic functions ranging from two to twenty-five dimensions, using both polynomial and kriging corrections from multiple starting points, totaling 170 cases. To summarize, the multifidelity quasi-Newton method used fewer high-fidelity function calls than monofidelity BFGS in 54% of cases, and the same number of calls in 9%. Compared to TRMM, the new approach required fewer high-fidelity evaluations 50% of the time, and the same number in 19% of cases. The new method also used up to 1.5 orders of magnitude fewer low-fidelity calls in 89% of cases, and the same number 10% of the time.

2. Parametric Geometry and Analysis Model Generation

A shared geometric representation of the vehicle is central to the multifidelity, multidisciplinary analysis and optimization, as illustrated in Figure 5. Using a single source ensures that the inputs given to each analysis are consistent and aids in the transfer of data between disciplines. This objective is achieved using CAPS.\textsuperscript{24}
Within CAPS exists a parametric, attributed model of the lambda wing vehicle, built using the Open-Source Constructive Solid Modeler (OpenCSM),\textsuperscript{34} which is in turn built upon the Engineering Geometry Aerospace Design System (EGADS).\textsuperscript{35} The attributes provide logical information required for the generation of analysis inputs. For example, attributes identify the vehicle skins where aeroelastic data transfers take place, symmetry planes for the application of boundary conditions, and bodies to which material properties should be applied. When a design parameter is changed, the geometry is regenerated, and analysis models (meshes, properties, etc.) may be requested for various disciplinary analyses. The inputs for these analyses are created via Analysis Interface Managers (AIMs), which manage the interpretation of the geometry into the required formats. At the time of writing, analyses demonstrated include structural FEA using two-dimensional elements and aerodynamics from panel methods up to Navier-Stokes.

The analysis model generation proceeds as follows. Using the current design parameters, the airfoil cross-sections and the planform shape are determined. Lofting these airfoils provides a solid body representing the outer mold line (OML). These same airfoils also provide the boundaries for defining mid-surface aerodynamic panel models. The CFD domain is generated by subtracting the OML solid from a bounding box. The internal structure results from intersecting the OML body with a grid representing the structural layout. The layout may have variable topology, though here the topology is held constant, and the shape follows the planform parameterization. The wing skins are extracted from the outer surface of the OML body.

3. Low-Fidelity Aeroelastic Modeling

The low-fidelity analysis is performed with the ASTROS\textsuperscript{25,26} package. ASTROS performs static, modal, and transient linear FEA, and has an internal aerodynamics capability for static and dynamic aeroelastic analyses. The aerodynamics model, Unified Subsonic and Supersonic Aerodynamic Analysis (USSAERO),\textsuperscript{36} generates pressures based on the superposition of sources and vortices over a panel representation of the vehicle, with a compressibility correction for both subsonic and supersonic flows. USSAERO simulates both lifting surfaces and non-lifting bodies, though the latter capability is not required for the current configuration. The transfer of loads and displacements between the two disciplines is handled using surface splines.\textsuperscript{37} The optimization utilizes the ASTROS static aeroelastic capability, specifying the angle of attack, vehicle shape, and structural gauges, and receiving the vehicle weight, finite element stresses and displacements, and lift and induced drag coefficients. A mesh convergence study for the aeroelastic cruise prediction is presented in Table 2 for the baseline configuration. Two additional configurations are considered by Bryson.\textsuperscript{38} The final models selected have approximately 2200 nodes and 1664 aerodynamic panels for the low-fidelity simulation.

<table>
<thead>
<tr>
<th>Table 2. Mesh convergence of low-fidelity simulation. Errors are with respect to finest mesh.</th>
</tr>
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<tbody>
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<td>FEA Nodes</td>
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<tr>
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</tr>
<tr>
<td>954</td>
</tr>
<tr>
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</tr>
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<td>2249</td>
</tr>
</tbody>
</table>

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4. High-Fidelity Aeroelastic Modeling

The high-fidelity analysis uses the internal aeroelasticity capability\textsuperscript{39} in the NASA FUN3D code.\textsuperscript{27, 28} The results presented here assume an inviscid fluid. FUN3D is a node-centered, implicit, upwind-differencing finite-volume solver.\textsuperscript{27, 39} The integrated aeroelastic analysis utilizes a modal structures representation computed by an external solver, in this case ASTROS. While the coupled aeroelastic equations are implemented for transient analysis,\textsuperscript{39} here the solution is time-marched to static equilibrium. The volume mesh deformation implements a linear elasticity analogy driven by the surface mesh displacements transferred from the structure.\textsuperscript{39} The initial grids are generated using AFLR\textsuperscript{40, 41} for the surface and volume. The transfer of mode shapes from the structural mesh to the fluid surface mesh is handled internally and automatically by CAPS without user intervention.\textsuperscript{12}

A grid convergence study is provided in Table 3 again for the baseline configuration. The meshes selected have approximately 800,000 CFD nodes and 2200 nodes for the FEA mesh. The finite element model is allowed to have rigid body motion about the wing box root to simulate free flight, though the rigid body modes are omitted from the CFD simulation. Eighteen mode shapes (not including the rigid body modes) are included in the dynamic analysis. Inviscid wall boundary conditions are applied to the wing outer mold line, and the symmetry plane is modeled with a symmetric boundary condition. Additional details of the numerical solution are provided by Bryson and Rumpfkeil.\textsuperscript{23}

![Table 3. Mesh convergence of high-fidelity simulation. Errors are with respect to finest mesh.](image)

![Figure 6. Typical convergence histories for aeroelastic CFD.](image)

Typical convergence histories are shown in Figure 6. Because finite differences are used for gradients, the solution must be deeply converged to ensure accurate sensitivities. After a study of twenty-nine different configurations, the solution was found to be highly converged after a non-dimensional simulation time of 1000, or approximately 650 time steps.

A finite-difference step size study was performed to identify the proper perturbation to calculate the gradients. The final step sizes selected for driving the optimization are presented in Table 4. The derivatives...
Figure 7 compares the starting and target planforms. The gradient norm convergence tolerance was set to solution, and level of iterative error in the high-fidelity solution.

These gradient errors are attributed to the finite-differencing of responses as expected. However, the derivatives are significantly non-zero, indicating that the optimizer would not find this point to be the minimum. Due to the constraints implicit in the shape parameterization, the wing-body bending was used to help prevent the superposition of mode shapes from leading to multiple local minima. Due to the constraints implicit in the shape parameterization, the y-coordinate was fixed at the outboard-wing root, as was the x-coordinate at the trailing edge break. The y-coordinate was also omitted from the tip trailing edge because it followed the position of the tip leading edge; hence, its error was not doubly counted. In summary, the inverse design objective function was

$$
\phi_{inv} = (x_0 - x_{0,targ}) + (x_0' - x_{0,targ}')^2 + (y_0 - y_{0,targ}) + (y_0' - y_{0,targ}')^2 \\
+ (z_0 - z_{0,targ}) + (z_0' - z_{0,targ}')^2 + (x_1 - x_{1,targ}) + (x_1' - x_{1,targ}')^2 \\
+ (y_1 - y_{1,targ}) + (y_1' - y_{1,targ}')^2 + (z_1 - z_{1,targ}) + (z_1' - z_{1,targ}')^2 \\
+ (x_2 - x_{2,targ}) + (x_2' - x_{2,targ}')^2 + (y_2 - y_{2,targ}) + (y_2' - y_{2,targ}')^2 \\
+ (z_2 - z_{2,targ}) + (z_2' - z_{2,targ}')^2 + (x_3 - x_{3,targ}) + (x_3' - x_{3,targ}')^2 + (y_3 - y_{3,targ}) + (y_3' - y_{3,targ}')^2 \\
+ (z_3 - z_{3,targ}) + (z_3' - z_{3,targ}')^2,
$$

where the root leading edge is denoted by a subscript 0, the root trailing edge by 1, the tip leading edge by 2, and the tip trailing edge by 3. The target design variables are tabulated in Table 5, along with the composite objective function was evaluated at the target point. The gradient norm is taken to be the infinity-norm of the gradient projected onto the side-constrained design space. When evaluated exactly at the target design, the composite objective function essentially vanishes as expected. However, the derivatives are significantly non-zero, indicating that the optimizer would not find this point to be the minimum. These gradient errors are attributed to the finite-differencing of responses containing numerical error. Though great effort was taken to reduce the error levels, the sources include the discretization error in the fluid and structural meshes, differences due to remeshing, residual error in modal solution, and level of iterative error in the high-fidelity solution.

Both the high- and multifidelity optimizers were started from the design space corner nearest the target. Figure 7 compares the starting and target planforms. The gradient norm convergence tolerance was set to

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>High-Fidelity Analysis</th>
<th>Low-Fidelity Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Step in Vars. Norm. [0,1]</td>
<td>Actual Step</td>
</tr>
<tr>
<td>Angle of Attack $\alpha$</td>
<td>$2.0 \times 10^{-6}$</td>
<td>$1.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Aspect Ratio $AR$</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$2.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>Sweep $\Delta$</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$2.5 \times 10^{-1}$</td>
</tr>
<tr>
<td>Taper Ratio $\lambda$</td>
<td>$1.0 \times 10^{-2}$</td>
<td>$3.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Skin Gauge $t_{skin}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$4.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Spar Gauge $t_{spar}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$4.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Rib Gauge $t_{rib}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$4.5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Table 5. Objective and gradient for inverse design of deformed shape, evaluated at the target point. The derivative listed for the composite objective is the projected gradient norm.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Target</th>
<th>Derivative $\frac{\partial}{\partial u_i} (\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AoA (deg)</td>
<td>0.5</td>
<td>$5.202 \times 10^{-7}$</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>3.5</td>
<td>$9.759 \times 10^{-1}$</td>
</tr>
<tr>
<td>Sweep Angle (deg)</td>
<td>50</td>
<td>1.713</td>
</tr>
<tr>
<td>Taper Ratio</td>
<td>0.3</td>
<td>$5.697 \times 10^{-2}$</td>
</tr>
<tr>
<td>Skin Gauge (mm)</td>
<td>12.5</td>
<td>$8.239 \times 10^{-5}$</td>
</tr>
<tr>
<td>Spar Gauge (mm)</td>
<td>12.5</td>
<td>$2.600 \times 10^{-6}$</td>
</tr>
<tr>
<td>Rib Gauge (mm)</td>
<td>12.5</td>
<td>$1.435 \times 10^{-7}$</td>
</tr>
<tr>
<td>Composite Objective</td>
<td>$2.602 \times 10^{-22}$</td>
<td>1.713</td>
</tr>
</tbody>
</table>

Figure 7. Planforms of target (black) and initial point (red) for inverse design.

$10^{-5}$, though in light of the results at the target point, it was not expected to be a reliable measure of convergence. It is included in the presentation of results only for completeness. A maximum budget of 20 high-fidelity evaluations was allotted. The initial trust region size was allowed to envelop the entire domain, and the multifidelity algorithm was set to revert to high-fidelity mode if the trust region fell below 0.001% for three consecutive iterations. This small limit was used to compare the performance of the high- and fully multifidelity processes in converging to the target point and did not come into effect during the optimization.

For the inverse design of the deformed shape, the multifidelity and high-fidelity optimizers followed the same path for the first ten function evaluations. This result was not surprising as the deformed coordinates were driven by the planform shape, and the CAPS framework ensured consistency of geometry between fidelity levels and disciplines. The convergence of the objective function and gradient norm is shown in Figure 8. Both the high- and multifidelity optimizers reduced the coordinate errors by four orders of magnitude. The difference in the objective convergence between the fifth and tenth evaluations was due to the multifidelity algorithm querying the high-fidelity model only once per iteration, while the high-fidelity algorithm required additional line search evaluations. Ultimately, the multifidelity result had a slightly smaller objective value with a gradient norm approximately one order of magnitude lower, but well above the convergence tolerance.

The errors in the normalized design variables from their targets are also plotted in Figure 8. Both optimizers most closely matched the shape parameters, which was expected since the wing coordinates were driven significantly more by the planform shape than the aeroelastic deformations. However, they had more difficulty in reducing the errors of the angle of attack and material gauges from their targets. This behavior was a result of the relatively small impact these variables had on the deformed shape and the inaccuracies of the finite difference gradients.

The numbers of low-fidelity evaluations and rejected points and the trust region size are plotted in Figure 9. These plots indicate that the corrected low-fidelity model performed remarkably well for this case, again attributable to the consistency of the geometry definition between fidelity levels. No steps were rejected until after the eleventh evaluation, when the optimizer began fine tuning the deformation. This was marked by an increase in low-fidelity line search calls and rejected designs and a steadily contracting trust region.
B. Optimization with Inequality Trim Constraint

The enforcement of equality constraints is known to slow the progress of gradient-based optimizers as they are forced to keep the violation small. An alternative was to replace the lift-trim condition with an inequality constraint, where excess lift was not penalized, and the drag minimization aspect of the problem was relied upon to force the constraint to be active. To evaluate the performance of the multifidelity approach for aero-structural design, optimizations were started from the two initial points listed in Table 6. Based on the analysis of the objective function, the first point was selected to be a point with low empty weight in an attempt to increase the influence of drag minimization. The second point was chosen nearer the center of the design space. The optimizations were compared using both the high-fidelity BFGS and multifidelity methods. The final designs are also provided in Table 6, and the planforms are shown in Figure 10. The gradient norm convergence tolerance was relaxed to $10^{-4}$ for these cases, but was not expected to be achieved given the
quality of the gradients. An allotment of 40 high-fidelity evaluations was given to each case, approximately ten days of runtime. The initial trust region was aggressively set to contain the entire design domain, and the minimum size was prescribed to be 0.1%. When the trust region fell below this limit for three consecutive iterations, the optimization reverted to high-fidelity mode.

Table 6. Initial and final designs for optimization with inequality lift trim constraint.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Start 1 Initial</th>
<th>Start 1 HF Final</th>
<th>Start 1 MF Final</th>
<th>Start 2 Initial</th>
<th>Start 2 HF Final</th>
<th>Start 2 MF Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>AoA (deg)</td>
<td>1.0</td>
<td>1.027</td>
<td>1.044</td>
<td>2.0</td>
<td>0.928</td>
<td>0.584</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>3.5</td>
<td>3.482</td>
<td>2.818</td>
<td>3.0</td>
<td>3.130</td>
<td>3.020</td>
</tr>
<tr>
<td>Sweep Angle (deg)</td>
<td>35.0</td>
<td>35.95</td>
<td>39.51</td>
<td>52.5</td>
<td>51.33</td>
<td>58.77</td>
</tr>
<tr>
<td>Taper Ratio</td>
<td>0.5</td>
<td>0.500</td>
<td>0.493</td>
<td>0.35</td>
<td>0.351</td>
<td>0.357</td>
</tr>
<tr>
<td>Skin Gauge (mm)</td>
<td>12.50</td>
<td>8.337</td>
<td>5.000</td>
<td>16.25</td>
<td>6.257</td>
<td>6.113</td>
</tr>
<tr>
<td>Spar Gauge (mm)</td>
<td>7.50</td>
<td>7.041</td>
<td>5.000</td>
<td>16.25</td>
<td>11.04</td>
<td>5.787</td>
</tr>
<tr>
<td>Rib Gauge (mm)</td>
<td>7.50</td>
<td>7.323</td>
<td>5.000</td>
<td>16.25</td>
<td>12.43</td>
<td>10.12</td>
</tr>
<tr>
<td>Objective</td>
<td>1.594</td>
<td>1.381</td>
<td>1.279</td>
<td>2.083</td>
<td>1.306</td>
<td>1.135</td>
</tr>
<tr>
<td>Lift Con</td>
<td>0.032</td>
<td>$6 \times 10^{-4}$</td>
<td>$-0.005$</td>
<td>$-0.355$</td>
<td>$-0.005$</td>
<td>$-0.004$</td>
</tr>
<tr>
<td>KS Skin</td>
<td>$-0.339$</td>
<td>0.021</td>
<td>$-0.073$</td>
<td>$-0.635$</td>
<td>0.021</td>
<td>0.025</td>
</tr>
<tr>
<td>KS Spar</td>
<td>$-0.481$</td>
<td>$-0.225$</td>
<td>$-0.252$</td>
<td>$-0.745$</td>
<td>$-0.337$</td>
<td>$-0.319$</td>
</tr>
<tr>
<td>KS Rib</td>
<td>$-0.398$</td>
<td>$-0.264$</td>
<td>$-0.492$</td>
<td>$-0.744$</td>
<td>$-0.519$</td>
<td>$-0.225$</td>
</tr>
</tbody>
</table>

Figure 10. Changes in planform shape via optimization with inequality lift trim constraint. Black lines are the initial shape, red lines the high-fidelity result, and blue lines the multifidelity result.

Figure 11 compares the performance of the multi- and high-fidelity approaches in reducing the penalty function and gradient norms, while Figure 12 illustrates reductions of the true objective. From the first initial design, both algorithms took the same starting step. After rejecting the second candidate step, the multifidelity optimizer reduced the penalty function by an additional seven percentage points, while the high-fidelity optimizer only achieved two points reduction. By the fourteenth evaluation, both optimizers were on par in terms of both penalty and objective function reductions. After this stage, the high-fidelity optimizer made no appreciable progress.

By the eighteenth high-fidelity evaluation, the trust region size, shown in Figure 13, had fallen below 0.1% for three consecutive iterations, and the multifidelity optimization was deemed to be making insufficient progress, though it had not expended its computational budget. To reiterate, the true objective function was equal between the two optimizers at this juncture, but the lift constraint was not sufficiently near zero to terminate the multifidelity optimization. (Recall that in this inequality-constrained formulation, the lift constraint must be active for the objective value to be valid.) Leveraging the approximate inverse Hessian maintained by the multifidelity algorithm, the optimization was resumed using high-fidelity data alone, and proceeded to drive the lift constraint to within 0.5% of violation. In the process, the multifidelity optimizer (in high-fidelity mode) reduced the objective function by and additional six percentage points, though the
total number of high-fidelity evaluations doubled. The design constraints are plotted in Figure 14.

From the second initial design, the multifidelity optimizer garnered a significant advantage in reducing both the penalty and objective functions in the first seven high-fidelity evaluations. The high-fidelity optimizer required seventeen evaluations to achieve comparable reductions, after which it made minimal progress. Within twenty high-fidelity evaluations, the multifidelity optimizer had reduced the objective function by 42%, compared to the high-fidelity optimizer’s 36%. However, the multifidelity progress ceased as the trust region size shrank below 0.1%. Similar to the first starting point, the lift constraint was not sufficiently active, and the optimization resumed as a high-fidelity process. These final iterations drove the trim constraint to its boundary and reduced the objective by an additional three percentage points.

High-fidelity function evaluations are only one aspect of the optimization cost; the expense of low-fidelity evaluations must also be considered. The numbers of low-fidelity evaluations and rejected candidate designs...
are plotted in Figure 13. For both initial designs, several low-fidelity calls were required during each line search, and many candidate points were rejected early on. In later iterations, the rate of increase in the number of low-fidelity evaluations and rejected designs slowed, indicating the optimizer found a more suitable low-fidelity correction and trust region size.

Considering the total high- and low-fidelity modeling cost, the return on investment of the optimization may be investigated in terms of penalty function reduction per processor-core-hour. The penalty function is used because it is the objective from the perspective of the optimizer and is guaranteed not to increase. On the other hand, the engineering objective (i.e., reciprocal range) may increase from iteration to iteration if it is balanced by constraint violation reductions. The approximate relative costs of the two fidelity levels are 600 processor-core-hours versus 1.6 processor-core-hours to compute the responses and finite-difference gradients. The cost of other overhead during the optimization is negligible. The marginal improvement per processor-core-hour highlights the improvement in each iteration for investing additional resources, plotted in Figure 15. Because the multifidelity algorithm will reject steps that increase the penalty function (skewing
Figure 14. Constraint functions for optimization with inequality lift trim constraint. Solid lines are high-fidelity results, and dashed lines are multifidelity results. The tip displacement, positive lift, and positive fuel mass constraints are omitted for clarity as they were inactive during the optimization and are enforced only for robustness of the analyses.

Figure 15. Return on investment in terms of marginal penalty function reduction per processor-core-hour for optimization with inequality lift trim constraint. Larger values show greater return on investment.

the results by producing zero improvement and making subsequent improvements artificially cheaper), the marginal return is plotted in terms of penalty-reducing iterations. That is, the cost of rejected steps are compounded into the next accepted step, showing the true cost of obtaining an actual reduction. Because the high-fidelity optimizer only returns iteration when the line search reduces the penalty function, the penalty-reducing iteration is the same as the true iteration. Comparing the marginal percentage improvement per core hour, the multifidelity process performed better for most iterations than the high-fidelity method, and was able to perform more iterations given the same high-fidelity budget. These comparisons show the benefit of using the cheaper, low-fidelity model for optimization.
Figures 16–18 plot the design variable convergence throughout the optimizations. The multifidelity optimizer was more successful than the high-fidelity approach in driving the material gauges to their lower bounds, except for the rib gauge in the second case where the rib stress constraint was active. The high-fidelity optimizer, on the other hand, was not able to exploit the stress margins to reduce the material gauges and, by proxy, the weight. Both high- and multifidelity optimizers swept the wing back slightly from the first starting design. From the second starting point, the high-fidelity optimizer left the sweep virtually unchanged, while the multifidelity process swept it back 6°. The aspect and taper ratios were largely unchanged, except for the first multifidelity design, which reduced the aspect ratio. This change led to a significant reduction in the skin stress constraint which was inactive in the final design. This result suggests additional iterations would reduce the empty weight and bring the constraint back to its boundary.
Figure 18. Angle of attack and aspect ratio for optimization with inequality lift trim constraint. Solid lines are high-fidelity results, and dashed lines are multifidelity results.

Figure 19. Pressure contours on the initial (left), high fidelity-optimized (center), and multifidelity-optimized (right) vehicles for inequality-constrained design (top—upper surface, bottom—lower surface).

Pressure contours over the initial as well as optimized designs are shown in Figure 19. In general, the low-pressure region near the lower-surface tip was enlarged from the starting points, which would have the effect of shifting lift inboard, reducing wing bending stresses and induced drag. The displacement contours in Figure 20 also show that the optimized wings exhibit more washout, again shifting lift inboard.

C. Optimization with Equality Trim Constraint

Because the lift-trim constraint is conceptually an equality constraint, it is desirable to include it as such in the optimization statement. The relaxation to an inequality constraint in Section B gave the optimizer more freedom in reducing the objective, but was problematic when the constraint was not active in the final design as observed in the multifidelity cases. Thus, the same design problem was reconsidered with a true equality lift constraint. The same starting points and termination criteria were used as in the inequality-constrained
Figure 20. Vertical displacements on the initial (left), high fidelity-optimized (center), and multifidelity-optimized (right) vehicles for inequality-constrained design.

Table 7. Initial and final designs for optimization with equality lift trim constraint.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Initial</th>
<th>HF Final</th>
<th>MF Final</th>
<th>Initial</th>
<th>HF Final</th>
<th>MF Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>AoA (deg)</td>
<td>1.0</td>
<td>0.996</td>
<td>1.003</td>
<td>2.0</td>
<td>0.911</td>
<td>0.947</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>3.5</td>
<td>2.746</td>
<td>2.872</td>
<td>3.0</td>
<td>2.598</td>
<td>2.606</td>
</tr>
<tr>
<td>Sweep Angle (deg)</td>
<td>35.0</td>
<td>37.60</td>
<td>3.708</td>
<td>52.5</td>
<td>50.30</td>
<td>48.37</td>
</tr>
<tr>
<td>Taper Ratio</td>
<td>0.5</td>
<td>0.478</td>
<td>0.497</td>
<td>0.35</td>
<td>0.399</td>
<td>0.398</td>
</tr>
<tr>
<td>Skin Gauge (mm)</td>
<td>12.50</td>
<td>5.000</td>
<td>5.000</td>
<td>16.25</td>
<td>5.000</td>
<td>5.000</td>
</tr>
<tr>
<td>Spar Gauge (mm)</td>
<td>7.50</td>
<td>5.000</td>
<td>5.000</td>
<td>16.25</td>
<td>5.000</td>
<td>5.308</td>
</tr>
<tr>
<td>Rib Gauge (mm)</td>
<td>7.50</td>
<td>5.000</td>
<td>5.000</td>
<td>16.25</td>
<td>14.10</td>
<td>6.446</td>
</tr>
<tr>
<td>Objective</td>
<td>1.594</td>
<td>1.272</td>
<td>1.268</td>
<td>2.083</td>
<td>1.259</td>
<td>1.246</td>
</tr>
<tr>
<td>Lift Con</td>
<td>0.034</td>
<td>0.013</td>
<td>0.016</td>
<td>−0.355</td>
<td>0.009</td>
<td>0.003</td>
</tr>
<tr>
<td>KS Skin</td>
<td>−0.339</td>
<td>−0.115</td>
<td>9 × 10⁻⁴</td>
<td>−0.635</td>
<td>0.001</td>
<td>−0.020</td>
</tr>
<tr>
<td>KS Spar</td>
<td>−0.481</td>
<td>−0.284</td>
<td>−0.194</td>
<td>−0.745</td>
<td>−0.245</td>
<td>−0.257</td>
</tr>
<tr>
<td>KS Rib</td>
<td>−0.398</td>
<td>−0.510</td>
<td>−0.428</td>
<td>−0.744</td>
<td>−0.718</td>
<td>−0.496</td>
</tr>
</tbody>
</table>

The initial and final designs are listed in Table 7, and the planforms are shown in Figure 21.

Figure 22 shows the relative performance of the multi- and high-fidelity optimizers in reducing the penalty function and gradient norm in terms of the number of high-fidelity function evaluations. As anticipated, the optimizers required significantly more line search evaluations to maintain constraint satisfaction. This situation was precisely what the multifidelity method was designed to ameliorate. While the high-fidelity approach required several line search evaluations to reduce the objective without violating the trim condition, the multifidelity optimizer only required one high-fidelity evaluation per iteration (or two, in the initial startup), relying more heavily on the corrected low-fidelity model.

Figure 23 shows the reduction in the true objective function for each case. From the first starting point, the multifidelity algorithm displayed a significant advantage over using high-fidelity data alone, achieving greater reductions with lower cost. However, after the eighteenth evaluation, the high-fidelity method produced better designs, while the trust region slowed the progress of the multifidelity approach. Once the optimizer reverted to high-fidelity mode when the trust region became too small, it proceeded to reduce the objective function to be slightly better than the result using the high-fidelity optimizer.
Figure 21. Changes in planform shape via optimization with equality lift trim constraint. Black lines are the initial shape, red lines the high-fidelity, and blue ones the multifidelity results.

Figure 22. Penalty function reduction and gradient norm convergence for optimization with equality lift trim constraint.

From the second starting point, the objective function convergence histories of the two optimizers crossed twice in the first ten evaluations, showing no clear advantage of one method over the other. After the fifteenth high-fidelity evaluation, the shrinking trust region hindered the multifidelity optimizer while the high-fidelity optimization continued to reduce the objective function. However, once the multifidelity optimizer reverted to high-fidelity mode after the twenty-fourth evaluation, it made additional improvements, ultimately producing a lower objective value than the true high-fidelity optimizer.

The evolution of the trust region size is plotted in Figure 24 along with the number of rejected design steps. Both multifidelity optimizations were started with an aggressive trust region spanning the entire design space. This size was suitable for the first two to five iterations, and the algorithm maintained it accordingly. The large trust region size indicates that the corrected low-fidelity model was sufficiently accurate to approximate the penalty function reductions in the early iterations. This fact is corroborated by the small number of rejected design steps. In later iterations, the trust region size was reduced steadily to find suitable limits where the low-fidelity model can be made to accurately match the high-fidelity data. This behavior corresponds to the alternating sequence of accepted-and-rejected designs.

Considering the marginal return on investment of each iteration, plotted in Figure 25, the multifidelity optimizer not only performed better during most of the optimization in both cases, but was also able to perform more penalty-reducing iterations given the same high-fidelity budget. Hence, accounting for
low-fidelity model cost, computational savings were achieved in most iterations by using the multifidelity approach.

The lift and stress constraints for these design processes are illustrated in Figure 26. From the first starting point, the high- and multifidelity optimizers satisfied the equality trim constraint to within 1.3% and 1.6%, respectively. Tighter enforcement could be obtained with additional iteration or by increasing the penalty weights, but the final values are similar enough for comparison. From the second starting point, the trim constraint was satisfied to less than 1%. In three of the four cases, the skin stress constraints were active, with the exception being the high-fidelity design from the first starting point. With additional iterations, the material gauges would likely be reduced, making the vehicle lighter and further stressing the skins. Similar improvements could also be made in the spars and ribs, for which the stress constraints were inactive in all the cases. The failure of the optimizer to drive these constraints to the active boundaries indicates that the behavior of the penalty function is difficult for the optimizers to navigate.

Figures 27–29 plot the convergence of the design variables. Overall, the optimizers made the most headway by driving down the material gauges to reduce empty weight. The skin and spar gauges were driven to, or near, their lower bounds in all four cases. From the second starting point, the high-fidelity optimizer found a search direction that reduced the penalty function but increased the rib gauge dramatically. With additional iteration, this design variable would likely be reduced, as in the other three cases.

From the first starting point, both optimizers increased sweep slightly from the onset, reducing the effect of wave drag. From the second starting point, both optimizers ultimately unswept the wing slightly to reduce weight and stress. Only small changes were made to the taper ratio, to which the responses were insensitive.

In all four cases, the optimizer reduced the aspect ratio to 2.6–2.9. This change was most pronounced once the stress constraints became active. By reducing the aspect ratio, the wing bending moment was reduced, and the root cross-section was made slightly thicker, both of which reduce stress, allowing thinner material to be used. In all cases, the vehicle trims for lift near 1° angle of attack.

The pressure contours over the initial and optimized designs are shown in Figure 30, and the displacement contours in Figure 31. Similar to the inequality-constrained designs, the lift distribution was shifted inboard, and outboard wing washout was increased, both of which reduced wing bending stresses and induced drag.

IV. Conclusions and Future Work

A new approach to multifidelity, gradient-based optimization has been demonstrated for vehicle aerostructural design. In the two inequality-constrained cases, the multifidelity process yielded superior objective function reductions compared to using high-fidelity data alone. However, the trust region slowed the
Figure 24. Number of low-fidelity evaluations and rejected steps and trust region size for optimization with equality lift trim constraint.

optimizer from driving the trim constraint to its active boundary, which was required by the objective formulation. Using the approximate inverse Hessian, the optimization was seamlessly continued in high-fidelity mode, successfully driving the constraint to its boundary and further reducing the objective function. In the equality-constrained cases, both the high- and multifidelity processes were successful in reducing the objective function while enforcing the active constraints. Initially, the multifidelity algorithm yielded better objective reductions for a given high-fidelity expenditure. However, as the trust region shrank, the high-fidelity optimizer surpassed the multifidelity performance. Hence, identifying when to transition between multi- and high-fidelity optimization is critical. Reverting the multifidelity algorithm to high-fidelity mode, both optimizers ultimately produced comparable designs, expending the allotted budget.

Considering the return on investment in terms of design improvement per computational hour, the multifidelity process showed an advantage over using high-fidelity data alone. On a marginal, iteration-by-iteration basis, the multifidelity process showed greater returns in most iterations, highlighting the ability to reduce the cost of optimization by using cheaper, corrected low-fidelity models. The multifidelity optimizer
was also able to produce more design-improving iterations given the same computational budget as the high-fidelity optimizer.

In future work, explicit handling of constraints must be considered in the optimization. Taking sequential quadratic programming as an example, the process decomposes in a manner very similar to the unconstrained, quasi-Newton optimization. First, the expected constrained-optimal point may be found based on the quadratic subproblem with linearized constraints. Second, a line search may be performed toward this point, using corrected low-fidelity data. An area for investigation is whether a single correction on the line-search merit function (typically the augmented Lagrangian) may be used, or if the objective and constraints
should be approximated separately and then combined into the merit function. The same decomposition
could also be studied using the quadratic penalty function and the current unconstrained optimizer.

The quality of the multidisciplinary gradients needs to be addressed on two fronts in future work. First,
tools for calculating coupled, analytic gradients should be implemented and verified within the analysis
framework. Such tools are the subject of research in several research groups, and may become available
though various collaborations. Second, analytic shape sensitivities from CAPS must be propagated from
the geometry through the mesh to the analysis. Typically, response sensitivities are propagated back to
the analysis mesh, and sensitivities of the mesh with respect to shape parameters are computed via finite
differences. The current gap in chaining the CAPS analytic shape sensitivities to the analysis is in remeshing
as the geometry changes. To compute a fully analytic gradient, either the meshing algorithm needs to be

Figure 27. Material gauges for optimization with equality lift trim constraint. Solid lines are high-fidelity
results, and dashed lines are multifidelity results.

Figure 28. Sweep angle and taper ratio for optimization with equality lift trim constraint. Solid lines are
high-fidelity results, and dashed lines are multifidelity results.
differentiated, or the analysis mesh must be mapped from one geometry to the next in a consistent fashion. Bridging this gap is an area also currently under research.

With the addition of adjoint gradients, large-scale optimization cases may be performed to study the scalability of the multifidelity algorithm by increasing the number of shape and sizing parameters and, consequently, the stress constraints. Different constraint aggregation techniques may also be investigated to ensure that the stress constraints are represented accurately, and that the behavior of the aggregation function near the constraint boundary does not hinder progress.
Figure 31. Vertical displacements on the initial (left), high fidelity-optimized (center), and multifidelity-optimized (right) vehicles for equality-constrained design.

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