Two-Sample, Independent Measures t Test

In this example 14 University of Dayton students were randomly assigned to either receive a context before hearing a passage (the context before condition, \( n_1 = 7 \)) or to receive no context (the no context condition, \( n_2 = 7 \)). Both groups then heard the same passage. They rated how well they comprehended the passage on a seven point scale (1 = no comprehension and 7 = complete comprehension) and then recalled the passage. The researcher wants to know receiving the context before hearing the passage improves recall of the passage relative to not receiving the context.

1. Step 1: Write the null and alternative hypotheses and specify the probability of making a Type I error:

   \[ H_0: \mu_{\text{context}} - \mu_{\text{no context}} \leq 0 \]

   \[ H_1: \mu_{\text{context}} - \mu_{\text{no context}} > 0 \]

   \( \alpha = .05 \)

2. Step 2: Sketch and determine the critical region:

   \[ df = (n_1 - 1) + (n_2 - 1) = (7 - 1) + (7 - 1) = 12 \]

   From a table of critical t scores with \( \alpha = .05 \), one-tailed, \( t_{\text{critical}}(12) = 1.782 \)

   (The critical t is positive because the critical region is concentrated in the right tail of the distribution.)
3. Step 3: Calculate the test statistic:

a. Open SPSS

b. Either type the data (see the last page for the data) or open a data set. The class data set (for homework) is available from <http://academic.udayton.edu/gregelvers/psy216/SPSS/216dataS11.sav> . Save the data file somewhere and open it with SPSS.

c. Analyze | Compare Means | Independent-Samples T Test  (this means to click on the Analyze menu item, then click on the Compare Means option in the drop down menu and then click on the Independent-Samples T Test option from the menu.)

d. Move the dependent variable (the variable measured by the researcher, recall) into the Test Variable(s) box. You can either drag the variable name into the box, or select the variable by clicking on it and then clicking on the arrow button between the two boxes.
e. Move the independent variable (the variable that the researcher manipulated; context in this example) to the Grouping Variable box.

f. We must tell SPSS which observations belong to each level (condition) of the independent variable. Click the Define Groups... button:
g. The people in the no context condition have a value of 1 for the context variable. Enter that value (1) into the Group 1 box. The people in the context before condition have a value of 2 for the context variable. Enter 2 into the Group 2 box:

h. Click the Continue button

i. Click the OK button

j. The SPSS output viewer will open

k. The first part of the output gives descriptive statistics for the dependent variable for each condition (level of the independent variable):

<table>
<thead>
<tr>
<th>Context</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recall No context</td>
<td>7</td>
<td>4.0000</td>
<td>2.23607</td>
<td>.84515</td>
</tr>
<tr>
<td>Context before</td>
<td>7</td>
<td>7.5714</td>
<td>2.63674</td>
<td>.99659</td>
</tr>
</tbody>
</table>
i. This tells us that for the no context condition there were 7 participants (in the N column of the No context row), that the sample mean ($\overline{X}$) is 4.000 (mean column and no context row), the sample standard deviation (s) is 2.236, and the standard error of the means ($s_m$) is 0.845 (which should and does equal $s / \sqrt{N} = 2.236 / \sqrt{7}$). Likewise, this tells us that for the context before condition there were 7 participants (in the N column of the context before row), that the sample mean ($\overline{X}$) is 7.571 (mean column and the context before row), the sample standard deviation (s) is 2.637, and the standard error of the means ($s_m$) is 0.997 (which should and does equal $s / \sqrt{N} = 2.637 / \sqrt{7}$).

m. The second part of the output gives us the value of the t-test:

<table>
<thead>
<tr>
<th>Independent Samples Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Recall</td>
</tr>
<tr>
<td>Equal variances assumed</td>
</tr>
<tr>
<td>Equal variances not assumed</td>
</tr>
</tbody>
</table>

n. SPSS performs two t-tests – one that assumes that the homogeneity of variances assumption is true (the “Equal variances assumed” row in the output) and one that assumes that the homogeneity of variances assumption is false (the “Equal variances not assumed” row in the output.) First, we must decide which of the two t-tests to use. That is, first we must decide whether the variances are likely to be equal or not. To make this decision we look at the portion of the output labeled Levene’s Test for Equality of Variances. Levene’s test tests the following hypotheses:

$H_0: \sigma_1^2 = \sigma_2^2$ \quad The variances are equal

$H_1: \sigma_1^2 \neq \sigma_2^2$ \quad The variances are not equal

Look at the p value (Sig. value) in the section of the output labeled Levene’s Test for Equality of Variances. The p value is .458. If $p \leq \alpha$ then you should reject $H_0$. 


otherwise you should fail to reject $H_0$. $p$ is larger than $\alpha$, so we should fail to reject $H_0$. That is, there is insufficient evidence in this set of data to suggest that the variances are not equal. In plain English, if $p \leq \alpha$, then assume that the variances are equal otherwise assume that the variances are not equal. Thus, we will use the $t$ test that assumes the variances are equal.

4. Step 4: Make a decision: Find the $p$ value for the $t$-test assuming equal variances. For this output, that value is .018. Notice that SPSS gave the $p$ value for a two tailed hypothesis, but our hypothesis is one tailed (directional). Divide the two tailed $p$ value by 2 to get the one tailed $p$ value. The one tailed $p$ value is $0.018 / 2 = 0.009$.

If the $p$ value is less than or equal to the $\alpha$ level, then you should reject $H_0$. Otherwise, you should fail to reject $H_0$. Because $p = 0.009$ and $\alpha = 0.05$, we reject $H_0$ and conclude that it is likely the case that having contextual information improves recall of the subsequent passage of text.

5. Cohen’s $d$ must be calculated by hand. Some of the values are from the SPSS output:

$$\text{Pooled variance} = s_p^2 = \frac{( df_1 \cdot s_1^2 + df_2 \cdot s_2^2 )}{( df_1 + df_2 )}$$
$$= \frac{((7 - 1) \cdot 2.63674^2 + (7 - 1) \cdot 2.23607^2)}{(7 - 1) + (7 - 1)}$$
$$= 5.9762$$

$$\text{Estimated value of Cohen’s } d = \frac{M_1 - M_2}{\sqrt{s_p^2}}$$
$$= \frac{(7.5714 - 4.0000)}{\sqrt{5.9762}}$$
$$= 1.461$$

This is a large effect (greater than .8)

6. $r^2$ must be calculated by hand. Both values for the formula can be gotten from the SPSS output:

$$r^2 = \frac{t^2}{(t^2 + df)} = \frac{-2.733^2}{(-2.733^2 + 12)} = .384$$

This is a large effect (greater than .25).

7. The 95% confidence interval for the difference of the means is calculated from:

$$\bar{X}_1 - \bar{X}_2 \pm t_{\text{critical}} \cdot s_{M1 - M2} = 7.5714 - 4.0000 \pm 1.782 \cdot 1.30671 = 1.243 \text{ to } 5.900$$

Since the confidence interval does not contain 0, $H_0$ is not likely to be true.
Steps 1 and 2 are identical to those used with SPSS.

Step 3: Calculate the test statistic:

<table>
<thead>
<tr>
<th></th>
<th>Context Before</th>
<th>No Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Σ</td>
<td>53</td>
<td>28</td>
</tr>
<tr>
<td>n</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>7.571</td>
<td>4</td>
</tr>
<tr>
<td>SS</td>
<td>41.714</td>
<td>30</td>
</tr>
</tbody>
</table>

$s_p^2 = (SS_1 + SS_2) / (df_1 + df_2)$

$= (41.714 + 30) / ((7 - 1) + (7 - 1))$

$= 5.976$

$s_{M1-M2} = \sqrt{s_p^2 / n_1 + s_p^2 / n_2}$

$= \sqrt{5.976 / 7 + 5.976 / 7}$

$= 1.307$

$t = (M_1 - M_2) / s_{M1-M2}$

$= (7.571 - 4.000) / 1.307$

$= 2.733$

Step 4: Decide:

If the observed or calculated value of t (= 2.733) is in the tail cut off by the critical t (from a table, 1.782; see step 2 above), then reject $H_0$, otherwise, fail to reject $H_0$. Reject $H_0$. It is likely the case that receiving a context before hearing a passage improves recall of the passage.