1. Problem 4 from the text

A researcher conducts a repeated-measures experiment using a sample of \( n = 8 \) subjects to evaluate the differences among four treatment conditions. If the results are examined with an ANOVA, what are the \( df \) values for the \( F \)-ratio?

\[
df_{\text{Between}} = k - 1 = 4 - 1 = 3 \\
df_{\text{Within}} = k \times (n - 1) = 4 \times (8 - 1) = 28 \\
df_{\text{Between Subjects}} = n - 1 = 4 - 1 = 3 \\
df_{\text{Error}} = df_{\text{Within}} - df_{\text{Between Subjects}} = 28 - 3 = 25
\]

The \( df \) of the \( F \)-ratio are 3 (\( df_{\text{Between}} \)) and 25 (\( df_{\text{Error}} \)).

2. Problem 6 from the text

A published report of a repeated-measures research study includes the following description of the statistical analysis. "The results show significant differences among the treatment conditions, \( F(2, 20) = 6.10, p < .01. \)"

a. How many treatment conditions were compared in the study?

\[
df_{\text{between-treatments}} = k - 1 \\
k = df_{\text{between-treatments}} + 1 \\
k = 2 + 1 = 3
\]

b. How many individuals participated in the study?

\[
df_{\text{within-treatment}} = df_{\text{between subjects}} + df_{\text{error}} \\
k \times (n - 1) = (n - 1) + 20 \\
3 \times (n - 1) = (n - 1) + 20 \\
2 \times (n - 1) = 20 \\
2n - 2 = 20 \\
2n = 22 \\
n = 11
\]

3. Problem 8 from the text

The following data were obtained from a repeated-measures study comparing two treatment conditions. Use a repeated-measures ANOVA with \( \alpha = .05 \) to determine whether there are significant mean differences between the two treatments.
Step 1: State the hypotheses and specify \( \alpha \)

\( H_0: \mu_1 = \mu_2 \)

\( H_1: \text{not } H_0 \) (at least one of the means are different)

\( \alpha = .05. \)

Step 2: Calculate the statistic (we will do the traditional second step later, once we have the dfs)

\[
SS_{\text{Total}} = \sum X^2 - \frac{G^2}{N} = 500 - \frac{80^2}{16} = 100
\]

\[
SS_{\text{Within}} = \sum SS_{\text{inside each treatment}} = 18 + 18 = 36
\]

\[
SS_{\text{Between}} = \sum \frac{T^2}{n} - \frac{G^2}{N} = \frac{24^2}{8} + \frac{56^2}{8} - \frac{80^2}{16} = 64
\]

\[
df_{\text{Total}} = N - 1 = 16 - 1 = 15
\]

\[
df_{\text{Within}} = \sum df = (8 - 1) + (8 - 1) = 14
\]

\[
df_{\text{Between}} = k - 1 = 2 - 1 = 1
\]

\[
SS_{\text{Between subjects}} = \sum \frac{P^2}{k} - \frac{G^2}{N}
\]
\[
= \frac{8^2}{2} + \frac{14^2}{2} + \frac{6^2}{2} + \frac{8^2}{2} + \frac{14^2}{2} + \frac{10^2}{2} + \frac{8^2}{2} + \frac{12^2}{2} - \frac{80^2}{16}
\]
\[
= 32
\]
\[ SS_{\text{Error}} = SS_{\text{Within}} - SS_{\text{Between subjects}} = 36 - 32 = 4 \]

\[ df_{\text{Between subjects}} = n - 1 = 8 - 1 = 7 \]

\[ df_{\text{Error}} = df_{\text{Within}} - df_{\text{Between subjects}} = 14 - 7 = 7 \]

\[ MS_{\text{Between}} = \frac{SS_{\text{Between}}}{df_{\text{Between}}} = \frac{64}{1} = 64 \]

\[ MS_{\text{Error}} = \frac{SS_{\text{Error}}}{df_{\text{Error}}} = \frac{4}{7} = 0.571 \]

\[ F = \frac{MS_{\text{Between}}}{MS_{\text{Error}}} = \frac{64}{0.571} = 112 \]

Step 3: Make a decision and state a conclusion

Consult a table of critical values of \( F \) with 1 (\( df_{\text{Between}} \)) and 7 (\( df_{\text{Error}} \)) degrees of freedom with \( \alpha = .05 \). The critical \( F \) equals 4.738. Because the calculated \( F \) is in the tail cut off by the critical \( F \), we can reject \( H_0 \) and conclude that it is likely that the treatment had an effect.

4. Enter the data from problem 9 into SPSS. Use SPSS to answer the following question: Do the data indicate significant mean differences among the three treatments? Give \( H_0 \), \( H_1 \) and \( \alpha \). Is this a one-tailed or two-tailed test? Use SPSS to compute \( \eta^2 \). Write a sentence or two in APA format that summarizes the results of the analysis.

The following data were obtained from a repeated-measures study comparing three treatment conditions.

<table>
<thead>
<tr>
<th>Person</th>
<th>I</th>
<th>II</th>
<th>II</th>
<th>Person Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>( P = 6 )</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>( P = 15 )</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>( P = 9 )</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>( P = 6 )</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>( P = 9 )</td>
</tr>
</tbody>
</table>

\[ M = 1 \quad M = 2 \quad M = 6 \]
\[ T = 5 \quad T = 10 \quad T = 30 \]
\[ SS = 6 \quad SS = 10 \quad SS = 10 \]

Step 1: State the hypotheses and specify \( \alpha \)

\( H_0: \mu_1 = \mu_2 = \mu_3 \)

\( H_1: \) not \( H_0 \)

\( \alpha = .05 \)

Two-tailed
Step 2: Do the calculations

In SPSS, switch to Variable View (click on the tab in the lower left, Ctrl-T or click on View | Variables)

Create three variables with the name Treatment1, Treatment2, and Treatment3

Switch to Data View (click on the tab in the lower left, Ctrl-T or click on View | Data)

Type the five observations for Treatment I into the Treatment1 column, the five observations for Treatment II into the Treatment2 column and the five observations for Treatment III into the Treatment3 column. Make sure that all of Person’s A’s data are in first row, all of Person’s B’s data are in the second row, etc.

Click on Analyze | General Linear Model | Repeated Measures

In the Repeated Measures Define Factor(s) dialog box, type Treatment into the Within-Subject Factor Name: field and 3 (the number of conditions / treatments) into the Number of Levels field

Click Add

Click Define

In the Repeated Measures dialog box, move the three treatment variables (Treatment1, Treatment2 and Treatment3) into the Within-Subjects Variables (Treatment): box

Click the Options button

In the Repeated Measures: Options dialog box, move the IV (Treatment) into the Display Means for: box

Check the checkboxes to the left of Descriptive statistics and Estimates of effect size

Click Continue

Click OK

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment 1</td>
<td>1.0000</td>
<td>1.22474</td>
<td>5</td>
</tr>
<tr>
<td>Treatment 2</td>
<td>2.0000</td>
<td>1.58114</td>
<td>5</td>
</tr>
<tr>
<td>Treatment 3</td>
<td>6.0000</td>
<td>1.58114</td>
<td>5</td>
</tr>
</tbody>
</table>
Tests of Within-Subjects Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig.</th>
<th>Partial Eta Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>treatment</td>
<td>Sphericity Assumed</td>
<td>70.000</td>
<td>2</td>
<td>35.000</td>
<td>35.000</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Greenhouse-Geisser</td>
<td>70.000</td>
<td>1.600</td>
<td>43.750</td>
<td>35.000</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Huynh-Feldt</td>
<td>70.000</td>
<td>2.000</td>
<td>35.000</td>
<td>35.000</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Lower-bound</td>
<td>70.000</td>
<td>1.000</td>
<td>70.000</td>
<td>35.000</td>
<td>.004</td>
</tr>
<tr>
<td>Error(treatment)</td>
<td>Sphericity Assumed</td>
<td>8.000</td>
<td>8</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Greenhouse-Geisser</td>
<td>8.000</td>
<td>6.400</td>
<td>1.250</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Huynh-Feldt</td>
<td>8.000</td>
<td>8.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lower-bound</td>
<td>8.000</td>
<td>4.000</td>
<td>2.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

HSD = q * \sqrt{(MS_{Error} / n)} = 4.043 * \sqrt{1 / 5} = 1.808

Consult a table of critical values of the Studentized range with 3 means (k = 3), 8 degrees of freedom (df_{Error} = 8) and \( \alpha = .05 \). The critical \( q = 4.043 \).

Any pair of means which are at least the HSD apart (1.808) are reliably different from each other.

\( H_0: \mu_{Treatment 1} = \mu_{Treatment 2} \)

\( H_1: \mu_{Treatment 1} \neq \mu_{Treatment 2} \)

\( M_{Treatment 1} - M_{Treatment 2} = 1 - 2 = -1 \not\Rightarrow \) Fail to reject \( H_0 \)

\( H_0: \mu_{Treatment 1} = \mu_{Treatment 3} \)

\( H_1: \mu_{Treatment 1} \neq \mu_{Treatment 3} \)

\( M_{Treatment 1} - M_{Treatment 3} = 1 - 6 = -5 \not\Rightarrow \) Reject \( H_0 \) (Ignore the sign – the means are at least the HSD apart whether the difference is positive or negative.)

\( H_0: \mu_{Treatment 2} = \mu_{Treatment 3} \)

\( H_1: \mu_{Treatment 2} \neq \mu_{Treatment 3} \)

\( M_{Treatment 2} - M_{Treatment 3} = 2 - 6 = -4 \not\Rightarrow \) Reject \( H_0 \)

Table 1 shows the mean and standard deviation for each treatment. The repeated measures analysis of variance revealed a significant effect of treatment on the dependent variable, \( F(2, 8) = 35.000, p < .001, MS_{error}= 1.000, \eta^2 = .897 \). Tukey HSD tests revealed that Treatment 3 was reliably different from Treatments 1 and 2, but that there is insufficient evidence to suggest that Treatments 1 and 2 are
reliably different from each other.

Table 1

*Means and Standard Deviations for Each Treatment*

<table>
<thead>
<tr>
<th>Treatment</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.22</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.58</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1.58</td>
</tr>
</tbody>
</table>

4. Problem 16 from the text

The following summary table presents the results from a repeated-measures ANOVA comparing three treatment conditions, each with a sample of \( n = 11 \) participants. Fill in the missing values in the table. *(Hint: Start with the \( df \) values).*

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between treatments</td>
<td>( 30 = 2 \times 15 )</td>
<td>( 2 = 3 - 1 )</td>
<td>( 15 = 3 \times 5 )</td>
</tr>
<tr>
<td>Within treatments</td>
<td>80</td>
<td>( 30 = 32 - 2 )</td>
<td></td>
</tr>
<tr>
<td>Between subjects</td>
<td>( 20 = 80 - 60 )</td>
<td>( 10 = 11 - 1 )</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>( 20 = 30 - 10 )</td>
<td>( 3 = 60 / 20 )</td>
</tr>
<tr>
<td>Total</td>
<td>( 110 = 30 + 80 )</td>
<td>( 32 = 3 \times 11 - 1 )</td>
<td></td>
</tr>
</tbody>
</table>

\[ df_{\text{Between treatments}} = \# \text{ of levels of the IV} - 1 = 3 - 1 = 2 \]
\[ df_{\text{Total}} = \# \text{ of observations} - 1 = \# \text{ of levels of the IV} \times \# \text{ of people per condition} - 1 = 3 \times 11 - 1 = 32 \]
\[ df_{\text{Within treatments}} = df_{\text{Total}} - df_{\text{Between treatments}} = 32 - 2 = 30 \]
\[ df_{\text{Between subjects}} = \# \text{ of participants} - 1 = 11 - 1 = 10 \]
\[ df_{\text{Error}} = df_{\text{Within treatments}} - df_{\text{Between subjects}} = 30 - 10 = 20 \]

\[ MS_{\text{Error}} = SS_{\text{Error}} / df_{\text{Error}} = 60 / 20 = 3 \]
\[ MS_{\text{Between treatments}} = F_{\text{Between treatments}} \times MS_{\text{Error}} = 5 \times 3 = 15 \]
5. Problem 19 from the text

A repeated-measures experiment comparing only two treatments can be evaluated with either a $t$ statistic or an ANOVA. As we found with the independent measures design, the $t$ test and the ANOVA produce equivalent conclusions, and the two test statistics are related by the equation $F = t^2$. The following data are from a repeated-measures study: (Problem 19 from the text)

<table>
<thead>
<tr>
<th>Subject</th>
<th>Treatment 1</th>
<th>Treatment 2</th>
<th>Difference</th>
<th>Person Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>+2</td>
<td>$P = 6$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>+2</td>
<td>$P = 4$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>10</td>
<td>+10</td>
<td>$P = 10$</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>3</td>
<td>+2</td>
<td>$P = 4$</td>
</tr>
</tbody>
</table>

$M = 1$
$T = 4$
$SS = 2$

$M = 5$
$T = 20$
$SS = 34$

$M = 4$
$T = 16$
$SS = 48$

$N = 8$
$G = 24$
$\Sigma X^2 = 140$

a. Use a repeated measures $t$ statistic with $\alpha = .05$ to determine whether the data provide evidence of a significant difference between the two treatments. (Caution: ANOVA calculations are done with the $X$ values, but for $t$ you use the difference scores.)

Step 1: State the hypotheses and specify $\alpha$

$H_0: \mu_D = 0$
$H_1: \mu_D \neq 0$
$\alpha = .05$

Step 2: Determine the critical region

$df = n - 1 = 4 - 1 = 3$
The critical $t$ for $df = 3$, $\alpha = .05$, two-tailed is $t = \pm 3.182$

Step 3: Calculate the statistic

$$s^2 = \frac{SS}{df} = \frac{48}{4 - 1} = 16$$

$$s_{MD} = \sqrt{\frac{s^2}{n}} = \sqrt{\frac{16}{4}} = 2$$

$$t = \frac{M_D - \mu_D}{s_{MD}} = \frac{4 - 0}{2} = 2$$
Step 4: Make a decision and conclusion

Because the calculated \( t \) (2) is not in the tails cut off by the critical \( t \) (3.182), one fails to reject \( H_0 \) and concludes that there is insufficient evidence to suggest that the treatment had an effect.

b. Use a repeated measures ANOVA with \( \alpha = .05 \) to evaluate the data. (You should find that \( F = t^2 \).)

Step 1: State the hypotheses and specify \( \alpha \)
\( H_0: \mu_1 = \mu_2 \)
\( H_1: \text{not } H_0 \) (at least one of the means are different)
\( \alpha = .05 \).

Step 2: Calculate the statistic (we will do the traditional second step later, once we have the \( df \)s)

\[
SS_{Total} = \sum X^2 - \frac{G^2}{N} = 140 - \frac{24^2}{8} = 68
\]

\[
SS_{Within} = \sum SS_{Inside each treatment} = 2 + 34 = 36
\]

\[
SS_{Between} = \sum \frac{T^2}{n} - \frac{G^2}{N} = \frac{4^2}{4} + \frac{20^2}{4} - \frac{24^2}{8} = 32
\]

\[
df_{Total} = N - 1 = 8 - 1 = 7
\]

\[
df_{Within} = \sum df = (4 - 1) + (4 - 1) = 6
\]

\[
df_{Between} = k - 1 = 2 - 1 = 1
\]

\[
SS_{Between subjects} = \sum \frac{P^2}{k} - \frac{G^2}{N} = \frac{6^2}{2} + \frac{4^2}{2} + \frac{10^2}{2} + \frac{4^2}{2} - \frac{24^2}{8} = 12
\]

\[
SS_{Error} = SS_{Within} - SS_{Between subjects} = 36 - 12 = 24
\]

\[
df_{Between subjects} = n - 1 = 4 - 1 = 3
\]
\[ df_{\text{Error}} = df_{\text{Within}} - df_{\text{Between subjects}} = 6 - 3 = 3 \]
\[ MS_{\text{Between}} = \frac{SS_{\text{Between}}}{df_{\text{Between}}} = \frac{32}{1} = 32 \]
\[ MS_{\text{Error}} = \frac{SS_{\text{Error}}}{df_{\text{Error}}} = \frac{24}{3} = 8 \]
\[ F = \frac{MS_{\text{Between}}}{MS_{\text{Error}}} = \frac{32}{8} = 4 \]

Step 3: Make a decision and state a conclusion

Consult a table of critical values of \( F \) with 1 (\( df_{\text{Between}} \)) and 3 (\( df_{\text{Error}} \)) degrees of freedom with \( \alpha = .05 \). The critical \( F \) equals 10.128 (10.128 = 3.182 * 3.182 where 3.182 is the critical \( t \) found above). Because the calculated \( F \) is not in the tail cut off by the critical \( F \), we fail to reject \( H_0 \) and conclude that there is insufficient evidence to suggest that the treatment had an effect.