Welcome to MTH 207 02
My Contact Information

My Name: Dr. Mashburn
Office: SC 313C
Phone: 229-2511
Email: joe.mashburn@udayton.edu
Office Hours: MTWF 10:00 – 10:50
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Basic Information

What you will do for this class:
Homework Assigned occasionally.
About 5 problems each.
No partial credit.
No late homework.
Quizzes Every Friday.
More conceptual.
Two lowest scores dropped.
No calculators.
No makeups.
Tests Three tests.
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Getting Help

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Math Department tutors: they are available in SC 307 from 6:00 to 10:00, Monday through Thursday.

SLS tutoring: I will email the schedule to you and post it on my web site as soon as I get it. SLS will post it on their site: http://learningsupport.udayton.edu/tutor.

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There are two types of statistics.

DEFINITION
Descriptive statistics utilizes numerical and graphical methods to look for patterns in a data set, to summarize the information revealed in a data set, and to present that information in a convenient form.

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This is an example of **descriptive statistics**.
Researchers analyzed the heights of over 2800 students from grades 1 through 6. The students were divided into three equal groups based on age (youngest third, middle third, oldest third) within each grade level. The researchers found that the oldest group had the shortest average height and that this phenomenon was due mainly to the 133 children who were held back a grade level. It was inferred from this that older boys within grades were relatively shorter than younger boys and that height is a factor in the decision to hold a child back.

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An experimental unit is an object about which we collect data.

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A population is the complete set of units that we are interested in studying.

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DEFINITION
A statistical inference is an estimate, prediction, or some other generalization about a population based on information obtained from a sample.
Example of Basic Terms

The makers of potato chip Brand A wants to be able to advertise that consumers prefer theirs over Brand B. They give a blind taste test to 1000 consumers in a nearby grocery store. Each tester is asked to state a preference for Brand A or Brand B.
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1. Describe the experimental unit.

Someone who eats potato chips.

2. Describe the population.

The set of people who eat potato chips.

3. Describe the variable of interest.

The preference between Brand A and Brand B.

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Reliability

One aspect of statistics that we will address later is the reliability of the inference.

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A measure of reliability is a statement about the degree of uncertainty associated with a statistical inference. A bound on the estimation error is a number that our estimation error is unlikely to exceed.

For example, techniques that we will learn later might indicate that the inference in our chip taste test is within 5% of the true preference of the population. If 57% of the sample prefers Brand A, then we can be reasonably confident that between 52% and 62% of the population prefers Brand A.
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- One or more variables that are to be investigated
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Types of Data

DEFINITION
Quantitative data are measurements that are recorded on a naturally occurring numerical scale.

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Qualitative data are measurements that cannot be measured on a natural numerical scale; they can only be classified into one of a group of categories.

Examples
1. GPA - Quantitative
2. Dining facility preferred - Qualitative
3. Rate from 1 (strongly agree) to 5 (strongly disagree) - Qualitative
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- Designed Experiment
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Example of Data Collection

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Another Example of Data Collection

Are men or women better at remembering where they left a misplaced item? 300 men and women participated in a study in which each person placed 20 common objects in a 12-room “house” on a computer screen. Thirty minutes later the subjects were asked to recall where they put each object. For each object a recall variable was measured as “yes” or “no”.

1. Identify the population of interest.
Adults

2. Identify the sample.
The 300 men and women involved in the study

3. What method was used to collect the data?
Designed experiment

4. What type of data was collected?
Qualitative

5. Does the study involve descriptive or inferential statistics?
Both. The results of the sample must be described before an inference can be made. The goal is the inference.
Another Example of Data Collection

Are men or women better at remembering where they left a misplaced item? 300 men and women participated in a study in which each person placed 20 common objects in a 12-room “house” on a computer screen. Thirty minutes later the subjects were asked to recall where they put each object. For each object a recall variable was measured as “yes” or “no”.

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Another Example of Data Collection

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1. Identify the population of interest. Adults
2. Identify the sample. The 300 men and women involved in the study
3. What method was used to collect the data? Designed experiment
4. What type of data was collected? Qualitative
5. Does the study involve descriptive or inferential statistics? Both. The results of the sample must be described before an inference can be made. The goal is the inference.
Types of Samples

**DEFINITION**

A *representative sample* exhibits characteristics typical of those possessed by the target population.
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**DEFINITION**
A *random sample* of $n$ experimental units is a sample selected from the population in such a way that all samples of size $n$ have an equal chance of being selected.
Types of Samples

DEFINITION
A representative sample exhibits characteristics typical of those possessed by the target population.

DEFINITION
A random sample of n experimental units is a sample selected from the population in such a way that all samples of size n have an equal chance of being selected.

Even a randomly generated sample may not be representative.
Biased Samples

DEFINITION
A sample is biased when it does not represent the desired characteristics of the population. It is not representative.
Biased Samples

**DEFINITION**

A sample is **biased** when it does not represent the desired characteristics of the population. It is not representative.

**DEFINITION**

*Selection bias* results when a subset of the experimental units in the population is excluded.
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Nonresponse bias results when the a survey or study is conducted in such a way that data on some subjects is not obtained.
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Selection bias results when a subset of the experimental units in the population is excluded.

DEFINITION
Nonresponse bias results when the survey or study is conducted in such a way that data on some subjects is not obtained.

DEFINITION
Measurement bias refers to inaccuracies in the values of the data recorded.
Frequency Tables of Qualitative Data

Data is collected on the majors of students in MTH 148. This is qualitative data, and falls into several categories or classes.
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Frequency Tables of Qualitative Data

Data is collected on the majors of students in MTH 148. This is qualitative data, and falls into several categories or classes.

**DEFINITION**

A *class* one of the categories into which qualitative data can be classified.
Frequency Tables of Qualitative Data

Data is collected on the majors of students in MTH 148. This is qualitative data, and falls into several categories or classes.

**DEFINITION**

A *class* one of the categories into which qualitative data can be classified.

We are interested in the number of each of the majors and the percentage of the students in each of the majors.
Frequency Tables of Qualitative Data

Data is collected on the majors of students in MTH 148. This is qualitative data, and falls into several categories or classes.

**DEFINITION**
A *class* one of the categories into which qualitative data can be classified.

We are interested in the number of each of the majors and the percentage of the students in each of the majors.

**DEFINITION**
The *class frequency* is the number of observations in the data set that fall into the class.
DEFINITION

The class relative frequency is the class frequency divided by the total number of observations in the data set. That is,

$$\text{class relative frequency} = \frac{\text{class frequency}}{n}$$
Graphs of Qualitative Data

We can also use graphs to represent and display data.
We can also use graphs to represent and display data.

**DEFINITION**

In a *bar graph* the classes of the qualitative variable are represented by bars, where the height of each bar is either the class frequency, class relative frequency, or class percentage.
Graphs of Qualitative Data

We can also use graphs to represent and display data.

**DEFINITION**

In a *bar graph* the classes of the qualitative variable are represented by bars, where the height of each bar is either the class frequency, class relative frequency, or class percentage. The vertical scale must start at 0 and the bars must all be the same width.
We can also use graphs to represent and display data.

**DEFINITION**

In a **bar graph** the classes of the qualitative variable are represented by bars, where the height of each bar is either the class frequency, class relative frequency, or class percentage. The vertical scale must start at 0 and the bars must all be the same width.

**DEFINITION**

In a **pie chart** the classes of the qualitative variable are represented by slices of a circle. The size of each slice is proportional to the class relative frequency.
Graphs of Qualitative Data

DEFINITION
A Pareto diagram is a bar graph with the classes of the qualitative variable arranged by height in descending order from left to right.
Graphs of Qualitative Data

DEFINITION
A Pareto diagram is a bar graph with the classes of the qualitative variable arranged by height in descending order from left to right. The line in a Pareto diagram measures the cumulative frequency or cumulative relative frequency.
Dot Plots and Stem-and-Leaf Displays

In a **dot plot** the numerical value of each measurement is represented by a dot on a horizontal scale. Each time a value is repeated, a dot is placed in a vertical line above that value.
Dot Plots and Stem-and-Leaf Displays

In a **dot plot** the numerical value of each measurement is represented by a dot on a horizontal scale. Each time a value is repeated, a dot is placed in a vertical line above that value. In a **stem-and-leaf display** the numerical value of the variable is partitioned into a “stem” and a “leaf”. The possible stems are listed in order in a column. The leaf for each quantitative measurement in the data set is placed in the corresponding stem row. Leaves for values with the same stem are listed in the increasing order horizontally.
Frequency Tables for Quantitative Variables

Requirements for Frequency Tables:

• Classes must be mutually exclusive
• Every measurement must be included in a class
• The table must include all the classes
• The classes must all have the same “width”
• The number of classes should be between 5 and 20
Frequency Tables for Quantitative Variables

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• The number of classes should be between 5 and 20
Terminology

- **Lower class boundary:** The smallest value that can belong to the class
- **Upper class boundary:** The largest value that can belong to the class
- **Class mark:** The number half-way between the lower and upper boundaries of the class
- **Class width:** The difference between the lower boundaries of two consecutive classes
  \[
  \text{class width} = \text{high score} - \text{low score}
  \]
  Round this number up if it is not an integer.
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$$\text{class width} = \frac{\text{high score} - \text{low score}}{\text{number of classes}}$$

Round this number *up* if it is not an integer.
Creating a Frequency Table

1. Decide on the number of classes.
Creating a Frequency Table

1. Decide on the number of classes. Small set, so use 5.
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
3. Find the class width.
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
3. Find the class width. \( \frac{99 - 46}{5} = \frac{53}{5} = 10.6 \). Use 11.
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
3. Find the class width.
4. Select as a starting point either the lowest value or a value slightly smaller. This is the first lower class boundary.

Determine the number of measurements that lie within each class.
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
3. Find the class width.
4. Select as a starting point either the lowest value or a value slightly smaller. This is the first lower class boundary. Use 46.
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
3. Find the class width.
4. Select as a starting point either the lowest value or a value slightly smaller. This is the first lower class boundary.
5. Add the class width to find the lower boundaries of the successive classes.
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
3. Find the class width.
4. Select as a starting point either the lowest value or a value slightly smaller. This is the first lower class boundary.
5. Add the class width to find the lower boundaries of the successive classes. 46, 57, 68, 79, 90
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
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6. The upper boundary of a class will be one less than the lower boundary of the next class.
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5. Add the class width to find the lower boundaries of the successive classes.
6. The upper boundary of a class will be one less than the lower boundary of the next class. 56, 67, 78, 89, 100
Creating a Frequency Table

1. Decide on the number of classes.
2. Create a stem-and-leaf display of the data.
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4. Select as a starting point either the lowest value or a value slightly smaller. This is the first lower class boundary.
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6. The upper boundary of a class will be one less than the lower boundary of the next class.
7. Determine the number of measurements that lie within each class.
## Creating a Frequency Table

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$46 \leq x \leq 56$</td>
<td>2</td>
<td>0.053</td>
</tr>
<tr>
<td>$57 \leq x \leq 67$</td>
<td>13</td>
<td>0.342</td>
</tr>
<tr>
<td>$68 \leq x \leq 78$</td>
<td>9</td>
<td>0.237</td>
</tr>
<tr>
<td>$79 \leq x \leq 89$</td>
<td>7</td>
<td>0.184</td>
</tr>
<tr>
<td>$90 \leq x \leq 100$</td>
<td>7</td>
<td>0.184</td>
</tr>
</tbody>
</table>
A histogram is a bar graph for quantitative variables.
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The classes used in a histogram are determined in the same way that they are for a frequency table.
A histogram is a bar graph for quantitative variables.

The classes used in a histogram are determined in the same way that they are for a frequency table.

We can use SPSS to generate histograms.
Summation Notation

DEFINITION

If \( x_1, x_2, \ldots, x_n \) represent \( n \) numbers then

\[
\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n.
\]
Summation Notation

**DEFINITION**

If $x_1, x_2, \ldots, x_n$ represent $n$ numbers then

$$\sum_{i=1}^{n} x_n = x_1 + x_2 + \cdots + x_n.$$ 

Let $x_1 = 1, x_2 = 2, x_3 = 9, x_4 = 2, x_5 = 5, x_6 = 3,$ and $x_7 = 6.$
Summation Notation

**DEFINITION**

If $x_1, x_2, \ldots, x_n$ represent $n$ numbers then

\[ \sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n. \]

Let $x_1 = 1, x_2 = 2, x_3 = 9, x_4 = 2, x_5 = 5, x_6 = 3,$ and $x_7 = 6.$

\[ \sum_{i=1}^{7} x_i = 1 + 2 + 9 + 2 + 5 + 3 + 6 = 28 \]
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$$\sum_{i=1}^{7} x_i = 1 + 2 + 9 + 2 + 5 + 3 + 6 = 28$$

$$\sum_{i=1}^{7} x_i^2 = 1^2 + 2^2 + 9^2 + 2^2 + 5^2 + 3^2 + 6^2 = 1 + 4 + 81 + 4 + 25 + 9 + 36 = 160$$
Summation Notation

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If $x_1, x_2, \ldots, x_n$ represent $n$ numbers then

$$\sum_{i=1}^{n} x_n = x_1 + x_2 + \cdots + x_n.$$ 

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$$\sum_{i=1}^{7} x_i = 1 + 2 + 9 + 2 + 5 + 3 + 6 = 28$$

$$\sum_{i=1}^{7} x_i^2 = 1^2 + 2^2 + 9^2 + 2^2 + 5^2 + 3^2 + 6^2 =$$

$$1 + 4 + 81 + 4 + 25 + 9 + 36 = 160$$

$$\sum_{i=1}^{7} (x_i - 1)^2 = 0^2 + 1^2 + 8^2 + 1^2 + 4^2 + 2^2 + 5^2 =$$

$$0 + 1 + 64 + 1 + 16 + 4 + 25 = 111$$
DEFINITION

If $x_1, x_2, \ldots, x_n$ represent $n$ numbers then

$$\sum_{i=1}^{n} x_n = x_1 + x_2 + \cdots + x_n.$$ 

Let $x_1 = 1, x_2 = 2, x_3 = 9, x_4 = 2, x_5 = 5, x_6 = 3$, and $x_7 = 6$.

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$$0 + 1 + 64 + 1 + 16 + 4 + 25 = 111$$

$$\left(\sum_{i=1}^{7} x_i\right)^2 = (1 + 2 + 9 + 2 + 5 + 3 + 6)^2 = 28^2 = 784$$
DEFINITION

The mean of a set of quantitative data is the sum of the measurements, divided by the number of measurements in the set. We will use $n$ to denote the number of measurements of the data set and $x$ as the variable representing the values of the data. The symbol $\bar{x}$ will represent the mean of a sample.

Formula for a Sample Mean

$$\bar{x} = \frac{\sum x}{n}$$

If the data set is \{1, 2, 9, 2, 5, 3, 6\} then

$$\bar{x} = \frac{\sum x}{n} = \frac{1 + 2 + 9 + 2 + 5 + 3 + 6}{7} = 4.$$
DEFINITION

The mean of a set of quantitative data is the sum of the measurements, divided by the number of measurements in the set.

Example:

If the data set is \{1, 2, 9, 2, 5, 3, 6\}, then 

\[
x = \frac{\sum x}{n} = \frac{1 + 2 + 9 + 2 + 5 + 3 + 6}{7} = 4.
\]
DEFINITION

The *mean* of a set of quantitative data is the *sum of the measurements, divided by the number of measurements in the set.*

We will use \( n \) to denote the number of measurements of the data set and \( x \) as the variable representing the values of the data. The symbol \( \bar{x} \) will represent the mean of a sample.
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The mean of a set of quantitative data is the sum of the measurements, divided by the number of measurements in the set.

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For larger data sets we can use SPSS.
Comparing Sample and Population Means

The sample mean can approximate the mean of the population, but they are not usually equal.
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- The larger the sample, the more likely $\bar{x}$ is a good approximation of $\mu$. 
Comparing Sample and Population Means

The sample mean can approximate the mean of the population, but they are not usually equal. We will use $\mu$ to denote the population mean.

- The larger the sample, the more likely $\bar{x}$ is a good approximation of $\mu$.
- The more spread out the data, the less likely $\bar{x}$ is a good approximation of $\mu$. 
Consider the set \{0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\}. Does this accurately reflect most of the measurements?

**Definition**

The median of a set of quantitative data is the number in the middle of the data when it is arranged in ascending order.

**Calculating the Median of a Sample**

1. Arrange the n measurements from smallest to largest.
2. If n is odd, then the median M is the middle number.
3. If n is even, then the median M is the mean of the two middle numbers.
Consider the set \( \{0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\} \).

\( \bar{x} = 10,000 \).
Consider the set \[\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\}\]. \[\bar{x} = 10,000\]. Does this accurately reflect most of the measurements?
Consider the set \( \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\} \).

\( \bar{x} = 10,000 \). Does this accurately reflect most of the measurements?

**DEFINITION**

The *median* of a set of quantitative data is the number in the middle of the data when it is arranged in ascending order.
Medians

Consider the set \(\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\}\). 
\(\bar{x} = 10,000\). Does this accurately reflect most of the measurements?

**DEFINITION**

*The median of a set of quantitative data is the number in the middle of the data when it is arranged in ascending order.*

**Calculating the Median of a Sample**
Consider the set \( \{0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\} \).

\( \bar{x} = 10,000 \). Does this accurately reflect most of the measurements?

**DEFINITION**

The *median* of a set of quantitative data is the number in the middle of the data when it is arranged in ascending order.

**Calculating the Median of a Sample**

1. **Arrange the \( n \) measurements from smallest to largest.**
Consider the set \( \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\} \). \( \bar{x} = 10,000 \). Does this accurately reflect most of the measurements?

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**Calculating the Median of a Sample**

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Consider the set \( \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 100000\} \). \( \bar{x} = 10,000 \). Does this accurately reflect most of the measurements?

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The *median* of a set of quantitative data is the number in the middle of the data when it is arranged in ascending order.

**Calculating the Median of a Sample**

1. Arrange the \( n \) measurements from smallest to largest.
2. If \( n \) is odd, then the median \( M \) is the middle number.
3. If \( n \) is even, then the median \( M \) is the mean of the two middle numbers.
Examples of Medians

Consider the set \{5, 2, 7, 2, 10, 1, 3\}.

First arrange the data from smallest to largest: 1, 2, 2, 3, 5, 7, 10. The median is 3, the middle number.

Now consider the set \{5, 2, 7, 10, 1, 3\}. When we arrange the values there is not a middle number: 1, 2, 3, 5, 7, 10. This time, the median is the mean of the two numbers in the middle: \((3 + 5)/2 = 4\).

We can use SPSS when finding the median of a large set.
Examples of Medians

Consider the set \{5, 2, 7, 2, 10, 1, 3\}. First arrange the data from smallest to largest: 1, 2, 2, 3, 5, 7, 10.
Examples of Medians

Consider the set \{5, 2, 7, 2, 10, 1, 3\}. First arrange the data from smallest to largest: 1, 2, 2, 3, 5, 7, 10. The median is 3, the middle number.

Now consider the set \{5, 2, 7, 10, 1, 3\}. When we arrange the values there is not a middle number: 1, 2, 3, 5, 7, 10. This time, the median is the mean of the two numbers in the middle: \((3 + 5)/2 = 4\). We can use SPSS when finding the median of a large set.
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Mean, Median, and Data Shape

A data set with mean 6.52 and median 5 produced the following histogram.
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This shape is called skewed right. The mean is always larger than (or to the right of) the median. There is always a long tail to the right.
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A data set with mean 14.48 and median 16 produced the following histogram.

![Histogram with mean and median marked]
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This shape is called symmetric.
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Mean, Median, and Data Shape

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DEFINITION
The *mode* of a data set is the measurement that occurs the most often.
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In the data set \{8, 7, 9, 6, 8, 10, 9, 9, 7\} the mode is 9.
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In the data set \{8, 7, 9, 6, 8, 10, 9, 9, 7\} the mode is 9. There can be more than one mode.
Example

The mean is 10.9, the median is 5.5, and the mode is 1.
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The mean is 55.6, the median is 56, and the mode is 57.
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Consider the two data sets \{10, 11, 12, 13, 87, 88, 89, 90\} and \{46, 47, 48, 49, 51, 52, 53, 54\}. 

**DEFINITION** 
The range of a quantitative data set is the largest minus the smallest measurement. 

The range of the first data set is \(90 - 10 = 80\) and that of the second is \(54 - 46 = 8\). 

But if we add the numbers 0 and 100 to both of the two data sets, then the means and medians remain 50, while the range of both sets is now 100.
Consider the two data sets \{10, 11, 12, 13, 87, 88, 89, 90\} and \{46, 47, 48, 49, 51, 52, 53, 54\}. They both have mean and median equal to 50.
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Variance

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$$\sum (x - \bar{x}) = (46 - 50) + (47 - 50) + (48 - 50) + (49 - 50) + (51 - 50) + (52 - 50) + (53 - 50) + (54 - 50) = -4 - 3 - 2 - 1 + 1 + 2 + 3 + 4 = 0$$
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DEFINITION

The sample variance of a sample of size $n$ is $s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$.

Example:

The sample variance of the data set $\{10, 11, 12, 13, 87, 88, 89, 90\}$ is $1695.4$.

The sample variance of the data set $\{46, 47, 48, 49, 51, 52, 53, 54\}$ is $8.7$. 

Shortcut Formula for the Sample Variance

$s^2 = \frac{\sum x^2 - (\sum x)^2}{n}$.
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\]
\[
\frac{1}{n-1}
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We can calculate similar values for the population.
**Standard Deviation**

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We can calculate similar values for the population.

**Symbols for Variance and Standard Deviation**

- $s^2 = \text{sample variance}$
- $s = \text{sample standard deviation}$
- $\sigma^2 = \text{population variance}$
- $\sigma = \text{population standard deviation}$
Chebyshev’s Rule

If $k > 1$ then the percentage of the data that lies within $k$ standard deviations of the mean is at least $1 - 1/k^2$. 

This rule applies to all data sets, regardless of shape. The interval that contains all values within $k$ standard deviations of the mean is $(x - ks, x + ks)$. So if the mean is 10 and the standard deviation is 2 then the values within two standard deviations of the mean are between 6 and 14 and those within three standard deviations of the mean are between 4 and 16.
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If a data set has mean 35 and standard deviation 3 and we randomly select a measurement from this set, is it likely that the measurement is 20?

No. There are 5 standard deviations between 20 and 35, so at least 96% of the values must be larger than 20.

At least what percentage of the data lies within between 29 and 41?

These numbers are 2 standard deviations from the mean, so the percentage is at least $1 - \frac{1}{4} = \frac{3}{4} = 75$.

If the range of the data is known to be 36, is it likely that the standard deviation is really 3?

No. The standard deviation is probably between $\frac{1}{4}$ and $\frac{1}{6}$ of the range.
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The data *must* be symmetric and bell-shaped in order to use the Empirical Rule.
The Empirical Rule

Suppose that a data set has mean 60 and standard deviation 10.
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1. What percentage of the data lie between 50 and 70?

   68%
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Suppose that a data set has mean 60 and standard deviation 10.

1. What percentage of the data lie between 50 and 70? 68%

2. What percentage of the data lie between 50 and 80? 81.5%

3. What percentage of the data lie between 40 and 50? 13.5%

4. What percentage of the data lie beyond 60? 50%

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DEFINITION
The *pth percentile* of a data set is the measurement *m* such that *p*% of the measurements fall below *m*. 
Percentiles

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The *pth percentile* of a data set is the measurement *m* such that *p*% of the measurements fall below *m*.

Note that \((100 - p)\)% of the measurements lie above *m*. 
The z-score allows us to compare measurements from different data sets by giving us the relative position of the measurements within their respective data sets.
**z-scores**

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**DEFINITION**

*The sample z-score for a measurement $x$ in a data set with sample mean $\bar{x}$ and sample standard deviation $s$ is*  

$$z = \frac{x - \bar{x}}{s}$$

*The population z-score for a measurement $x$ in a population with population mean $\mu$ and population standard deviation $\sigma$ is*  

$$z = \frac{x - \mu}{\sigma}$$
Let $S_1$ be a sample with mean $\bar{x}_1 = 64$ and standard deviation 7. Let $S_2$ be a sample with mean 75 and standard deviation 10.

The $z$-score of 82 is
$$z(82) = \frac{82 - 64}{7} = 2.57.$$  
The $z$-score of 95 is
$$z(95) = \frac{95 - 75}{10} = 2.$$  
So 82 is a higher percentile than 95.
Let $S_1$ be a sample with mean $\bar{x}_1 = 64$ and standard deviation 7. Let $S_2$ be a sample with mean 75 and standard deviation 10.

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**Interpretation of $z$-scores for Bell-Shaped Distributions**

1. Approximately 68% of the measurements have $z$-scores between $-1$ and $1$.
2. Approximately 95% of the measurements have $z$-scores between $-2$ and $2$.
3. Approximately 99.7% of the measurements have $z$-scores between $-3$ and $3$.
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• Bad measurements
• Failing to include the variability of the sample
• Distorted graphs
  • y-axis begun at a number other than 0
  • Some parts of the graph are visually distorted
Distorting the Truth with Descriptive Statistics
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Events

DEFINITION
An experiment is an act or process of observation that leads to a single outcome. Some examples are: flipping a coin, rolling a die, picking a card, choosing a marble, asking a voter for an opinion.

DEFINITION
A simple event is a single outcome of an experiment. If a coin is flipped three times then the simple events are HHH, HHT, HTH, THH, HTT, THT, TTH, and TTT.
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Events and Sample Spaces

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An *event* is any collection of outcomes or simple events of an experiment.

In the experiment of flipping the coin 3 times, the event could be that we get 2 tails. The event consists of the outcomes $TTH$, $THT$, and $HTT$ and can be written as $\{TTH, THT, HTT\}$. 

**DEFINITION**

The sample space of an experiment is the set of all possible outcomes.

If the experiment is flipping a coin twice, then the sample space is $\{HH, HT, TH, TT\}$.

If the experiment is rolling a die, then the sample space could be written as $\{1, 2, 3, 4, 5, 6\}$. 
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DEFINITION

If an experiment has a sample space $S$ with $n(S)$ elements, each of which is equally likely, and an event $E$ contains $n(E)$ of these outcomes, then the probability of $E$ is $\frac{n(E)}{n(S)}$. 

The probability of getting a head when we toss a coin is $\frac{1}{2}$. The probability of getting 2 heads when we toss a coin 3 times is $\frac{3}{8}$. The probability of rolling a die and getting a prime number is $\frac{3}{6}$. The probability of choosing a card and getting a face card is $\frac{3}{13}$. 

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Probabilities
The probability of an event must lie between 0 and 1, that is, $0 \leq P(E) \leq 1$. 

Flip a coin 100 times and let $E = \text{at least one head}$. Let $F = \text{all tails}$. Then $P(E) = 1 - P(F) = 1 - \left(\frac{1}{2}\right)^{100} = \left(\frac{2}{2} - \frac{1}{2}\right)^{100}$. 


Probabilities

Probability Rules

1. The probability of an event must lie between 0 and 1, that is, \(0 \leq P(E) \leq 1\).

2. The sum of the probabilities of all simple events is 1.

Flip a coin 100 times and let \(E\) = at least one head. Let \(F\) = all tails. Then \(P(E) = 1 - P(F) = 1 - (1/2^{100}) = (2^{100} - 1)/2^{100}\).
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Combining Simple Events with “or”

The following table classifies students in a class by year and by type of major.

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\[ P(Y_1 \text{ or } Y_2) = \frac{11 + 14}{41} = \frac{11}{41} + \frac{14}{41} = P(Y_1) + P(Y_2) \]
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\[
P(Y2 \text{ or } EDU) = \frac{14 + 5}{41} = \frac{14 + 9 - 4}{41}
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\[
P(Y2 \text{ or } EDU) = \frac{14 + 5}{41} = \frac{14 + 9 - 4}{41} = \frac{14}{41} + \frac{9}{41} - \frac{4}{41}
\]
Combining Simple Events with “or”

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</tr>
</tbody>
</table>

\[ P(Y2 \text{ or } \text{EDU}) = \frac{14 + 5}{41} = \frac{14 + 9 - 4}{41} = \frac{14}{41} + \frac{9}{41} - \frac{4}{41} \]

\[ = P(Y2) + P(\text{EDU}) - P(Y2 \text{ and } \text{EDU}) \]
Addition Rule

\[ P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F) \]
The Addition Rule

Addition Rule

\[ P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F) \]

The book writes this as \( P(E \cup F) = P(E) + P(F) - P(E \cap F) \).
The Addition Rule

Addition Rule
\[ P(\text{E or F}) = P(\text{E}) + P(\text{F}) - P(\text{E and F}) \]
The book writes this as \( P(\text{E} \cup \text{F}) = P(\text{E}) + P(\text{F}) - P(\text{E} \cap \text{F}). \)

**DEFINITION**
*Two events are mutually exclusive if they cannot occur simultaneously.*
The Addition Rule

Addition Rule

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The book writes this as \( P(E \cup F) = P(E) + P(F) - P(E \cap F) \).

**DEFINITION**

*Two events are mutually exclusive if they cannot occur simultaneously.*

If E and F are mutually exclusive then \( P(E \text{ and } F) = 0 \).
Addition Rule

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$$

The book writes this as $$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$.

**DEFINITION**

*Two events are mutually exclusive if they cannot occur simultaneously.*

If E and F are mutually exclusive then $$P(E \text{ and } F) = 0$$. Then $$P(E \text{ or } F) = P(E) + P(F)$$. 
Examples of the Addition Rule

1. \( P(H \text{ or } S) \)
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1. \( P(H \text{ or } S) = P(H) + P(S) - P(H \text{ and } S) \)
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3. \( P(H \text{ or } K) \)
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4. \( P(F \text{ or } B) = P(F) + P(B) - P(F \text{ and } B) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13} \)
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5. \[ P(F \text{ or Not Even}) \]
Examples of the Addition Rule

1. \( P(H \text{ or } S) = P(H) + P(S) - P(H \text{ and } S) = \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2} \)

2. \( P(R \text{ or } S) = P(R) + P(S) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \)

3. \( P(H \text{ or } K) = P(H) + P(K) - P(H \text{ and } K) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} \)

4. \( P(F \text{ or } B) = P(F) + P(B) - P(F \text{ and } B) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52} \)

5. \( P(F \text{ or Not Even}) = \frac{12}{52} + \frac{36}{52} - \frac{12}{52} = \frac{36}{52} \)
Complementary Events

DEFINITION

The complement of an event $A$ is the event that $A$ does not occur.
DEFINITION

The *complement* of an event $A$ is the event that $A$ does not occur. The complement of $A$ is denoted $A^C$. 

Consider, again, the standard deck of cards. 

$$P(F_C) = 1 - P(F) = \frac{10}{13}$$
DEFINITION

The complement of an event $A$ is the event that $A$ does not occur.

The complement of $A$ is denoted $A^C$. $A^C$ is the collection of all simple events not in $A$. 

Consider, again, the standard deck of cards. 

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Consider, again, the standard deck of cards.

$P(F^C) = 1 - P(F)$
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The complement of an event $A$ is the event that $A$ does not occur.

The complement of $A$ is denoted $A^C$. $A^C$ is the collection of all simple events not in $A$.

Consider, again, the standard deck of cards.

$$P(F^C) = 1 - P(F) = 1 - \frac{3}{13} = \frac{10}{13}$$
Combining Simple Events with “and”

Roll a die and flip a coin. The sample space is given below.

1H 2H 3H 4H 5H 6H 1T 2T 3T 4T 5T 6T

P(4 and T) = \frac{1}{12}

P(4) \cdot P(T) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}

Suppose a bag contains 3 red and 2 white marbles. Pick a marble, replace it, and pick a second one.

P(R and R) = \frac{9}{25}

P(R) \cdot P(R) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}

This time, pick a marble and do not replace it before picking the second.

P(R and R) = \frac{3}{10}

The book uses “A ∩ B” in place of “A and B”.
Combining Simple Events with “and”

Roll a die and flip a coin. The sample space is given below.

1H 2H 3H 4H 5H 6H 1T 2T 3T 4T 5T 6T

\[ P(4 \text{ and } T) = \frac{1}{12} \]
Combining Simple Events with “and”

Roll a die and flip a coin. The sample space is given below.

\[
1H \ 2H \ 3H \ 4H \ 5H \ 6H \ 1T \ 2T \ 3T \ 4T \ 5T \ 6T
\]

\[
P(4 \text{ and } T) = \frac{1}{12} \quad P(4) \cdot P(T) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}
\]

Suppose a bag contains 3 red and 2 white marbles. Pick a marble, replace it, and pick a second one.

\[
P(R \text{ and } R) = \frac{9}{25}
\]

\[
P(R) \cdot P(R) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}
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This time, pick a marble and do not replace it before picking the second.

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\[1H \ 2H \ 3H \ 4H \ 5H \ 6H \ 1T \ 2T \ 3T \ 4T \ 5T \ 6T\]

\[P(4 \text{ and } T) = \frac{1}{12} \quad P(4) \cdot P(T) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}\]

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\[P(R \text{ and } R) = \frac{3}{10}\]

The book uses “A \cap B” in place of “A and B”.
Independent Events

When we rolled the die and flipped the coin, the two events had no affect on one another. When we replaced the marble, the choice made on the first marble had no affect on the choice of the second marble. But when the marble was not replaced, the choice of the first marble did affect the choice of the second.
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**DEFINITION**

Two events are said to be independent if the occurrence of one is unaffected by the occurrence of the other. If two events are not independent then they are said to be dependent.
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**DEFINITION**

*\( B \mid A \) means “\( B \) given “\( A \)”. It refers to the occurrence of event \( B \) given that event \( A \) has occurred.*
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Two events are said to be independent if the occurrence of one is unaffected by the occurrence of the other. If two events are not independent then they are said to be dependent.

**DEFINITION**

$B|A$ means “$B$ given “$A$”. It refers to the occurrence of event $B$ given that event $A$ has occurred.

If $A$ and $B$ are independent then $P(A|B) = P(A)$ and $P(B|A) = P(B)$. 
Multiplication Rule

Multiplication Rule

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]
Multiplication Rule

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

If $A$ and $B$ are independent then $P(A \text{ and } B) = P(A) \cdot P(B)$. 

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

$P(RWW) = P(R) \cdot P(W) \cdot P(W) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = \frac{144}{10^3}.$

Let $E$ be the event that two out of the three marbles are white.

$P(E) = P(RWW \text{ or } WRW \text{ or } WWR) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} + \frac{6}{10} \cdot \frac{4}{10} \cdot \frac{6}{10} + \frac{6}{10} \cdot \frac{6}{10} \cdot \frac{4}{10} = \frac{432}{10^3}.$
Multiplication Rule

Multiplication Rule

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

If A and B are independent then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

1. \( P(RWW) = P(R) \cdot P(W) \cdot P(W) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = \frac{144}{1000} = \frac{36}{250} \)

2. Let \( E \) be the event that two out of the three marbles are white. \( P(E) = P(RWW \text{ or } WRW \text{ or } WWR) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} + \frac{6}{10} \cdot \frac{4}{10} \cdot \frac{6}{10} + \frac{6}{10} \cdot \frac{6}{10} \cdot \frac{4}{10} = \frac{36}{100} + \frac{36}{100} + \frac{36}{100} = \frac{108}{100} = \frac{27}{25} \).
Multiplication Rule

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

If A and B are independent then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

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Multiplication Rule

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

If $A$ and $B$ are independent then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

1. $P(RWW) = P(R)P(W)P(W)$
Multiplication Rule

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

If A and B are independent then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

\[ P(RWW) = P(R)P(W)P(W) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = \frac{4 \cdot 6 \cdot 6}{10 \cdot 10 \cdot 10} \]
Multiplication Rule

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

If A and B are independent then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

1. \( P(RWW) = P(R)P(W)P(W) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = .144 \)
Multiplication Rule

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2. Let \( E \) be the event that two out of the three marbles are white.
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\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

If A and B are independent then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

1. \( P(RWW) = P(R)P(W)P(W) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = 0.144 \)

2. Let \( E \) be the event that two out of the three marbles are white.
   \[ P(E) \]
Multiplication Rule

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

If \( A \) and \( B \) are independent then \( P(A \text{ and } B) = P(A) \cdot P(B) \).

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

1. \( P(RWW) = P(R)P(W)P(W) = \frac{4}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = .144 \)

2. Let \( E \) be the event that two out of the three marbles are white.
   \( P(E) = P(RWW \text{ or } WRW \text{ or } WWR) \)
Multiplication Rule

Multiplication Rule

\[ P(A \text{ and } B) = P(A) \cdot P(B|A) \]

If A and B are independent then \[ P(A \text{ and } B) = P(A) \cdot P(B). \]

Suppose a bag contains 4 red and 6 white marbles. Three marbles are randomly selected with replacement.

1. \[ P(\text{RWW}) = P(\text{R})P(\text{W})P(\text{W}) = \frac{4}{10} \frac{6}{10} \frac{6}{10} = .144 \]

2. Let \( E \) be the event that two out of the three marbles are white.

\[ P(E) = P(\text{RWW or WRW or WWR}) \]
\[ = \frac{4}{10} \frac{6}{10} \frac{6}{10} + \frac{6}{10} \frac{4}{10} \frac{6}{10} + \frac{6}{10} \frac{6}{10} \frac{4}{10} = .432 \]
Multiplication Rule

Now suppose that the three marbles are selected without replacement.

\[ P(RWW) = P(R)P(W|R)P(W|RW) = \frac{1}{167} \]

If \( E \) is the event that at least one of the 3 marbles is red then
\[ P(E) = 1 - P(E^C) = 1 - P(WWW) = 1 - \frac{6}{167} = \frac{161}{167} = 0.9664 \]
Now suppose that the three marbles are selected without replacement.

1. \( P(RWW) \)
Multiplication Rule

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1. \( P(RWW) = P(R)P(W|R)P(W|RW) \)
Now suppose that the three marbles are selected without replacement.

\[
P(RWW) = P(R)P(W|R)P(W|RW) = \frac{4 \cdot 6 \cdot 5}{10 \cdot 9 \cdot 8}
\]
Now suppose that the three marbles are selected without replacement.

\[ P(RWW) = P(R)P(W|R)P(W|RW) = \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = .167 \]
Now suppose that the three marbles are selected without replacement.

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Now suppose that the three marbles are selected without replacement.

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Multiplication Rule

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Multiplication Rule

Now suppose that the three marbles are selected without replacement.

1. $P(RWW) = P(R)P(W|R)P(W|RW) = \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} = .167$

2. If $E$ is the event that at least one of the 3 marbles is red then
   $P(E) = 1 - P(E^C) = 1 - P(WWWW) = 1 - \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = .833$
Conditional Probability

We can use the Multiplication Rule to find the value of $P(A|B)$. 
Conditional Probability

We can use the Multiplication Rule to find the value of \( P(A \mid B) \).

\[
P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}
\]
Let $A$ be the event that a student is a junior, and let $B$ be the event that a student is in SCI.

\[
P(A | B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{7}{41} \div \frac{16}{41} = \frac{7}{16}
\]

$P(A) = \frac{13}{41}$

$P(A | B) \neq P(A)$ so $A$ and $B$ are dependent.
Let A be the event that a student is a junior, and let B be the event that a student is in SCI.

Let \( P(A \mid B) \) be the conditional probability of A given B.
Let A be the event that a student is a junior, and let B be the event that a student is in SCI.

\[ P(A|B) \]
Let $A$ be the event that a student is a junior, and let $B$ be the event that a student is in SCI.

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P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{7}{41} \quad \text{and} \quad \frac{16}{41}
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Let $A$ be the event that a student is a junior, and let $B$ be the event that a student is in SCI.

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P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{7}{41} \frac{41}{16} = \frac{7}{16}
\]
Conditional Probability

<table>
<thead>
<tr>
<th>Year</th>
<th>ART</th>
<th>EDU</th>
<th>ENG</th>
<th>SCI</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>14</td>
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<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>9</td>
<td>1</td>
<td>16</td>
<td>41</td>
</tr>
</tbody>
</table>

Let A be the event that a student is a junior, and let B be the event that a student is in SCI.

\[
P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{7}{41} = \frac{7}{16}
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\[
P(A)
\]
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Let $A$ be the event that a student is a junior, and let $B$ be the event that a student is in SCI.

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\]

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P(A) = \frac{13}{41}
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$P(A|B) \neq P(A)$ so $A$ and $B$ are dependent.
DEFINITION
A random variable is a variable that assumes numerical values associated with the random outcomes of an experiment, where one and only one value is assigned to each possible outcome.
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The experiment could be to randomly choose 5 students from the class and let $x$ be the number that are enrolled in the School of Education. The possible values of $x$ are 0, 1, 2, 3, 4, and 5.
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*Random variables that can assume a countable number of values are called *discrete*.**
Random Variables

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Random variables that can assume a countable number of values are called *discrete*.

**DEFINITION**
Random variables that can assume any value on some interval of real numbers are called *continuous*. 
Probability Distributions

Suppose that a coin is flipped 2 times and $x$ is the number of heads. The sample space is $\{HH, HT, TH, TT\}$. 

$P(x=0) = \frac{1}{4}$

$P(x=1) = \frac{2}{4} = \frac{1}{2}$

$P(x=2) = \frac{1}{4}$

This defines a function called a probability distribution. We can also draw a histogram for it.
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There is an important connection between the area in a probability distribution histogram and probabilities.
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<table>
<thead>
<tr>
<th>$x$</th>
<th>$p(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.45</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th></th>
<th>( p(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.20</td>
</tr>
<tr>
<td>2</td>
<td>.10</td>
</tr>
<tr>
<td>3</td>
<td>.45</td>
</tr>
<tr>
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<td>.25</td>
</tr>
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</table>
Probability Distributions
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The probability distribution of a discrete random variable is a function, graph, or table that specifies the probability of each possible value that the random variable can assume.
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Requirements for the Probability Distribution of a Discrete Random Variable

\[
p(x) \geq 0 \quad \text{for all } x
\]

\[
\sum p(x) = 1
\]
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2. \( \sum p(x) = 1 \)

Determine whether the following functions are probability distributions.

1. \( p(x) = x \) for \( x = 0, 1, 2, 3 \)
   - No
2. \( p(x) = x^2 \) for \( x = 0, 1, 2, 3, 4, 5, 6, 7 \)
   - Yes
Probability Distributions

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\[ p(x) = \frac{x}{7} \text{ for } x = 0, 1, 2, 3 \]
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No
Probability Distributions

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The probability distribution of a discrete random variable is a function, graph, or table that specifies the probability of each possible value that the random variable can assume.

Requirements for the Probability Distribution of a Discrete Random Variable

1. $p(x) \geq 0$ for all $x$
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Determine whether the following functions are probability distributions.

$p(x) = \frac{x}{7}$ for $x = 0, 1, 2, 3$ \hspace{1cm} No

$p(x) = \frac{x}{28}$ for $x = 0, 1, 2, 3, 4, 5, 6, 7$
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$p(x) = \frac{x}{28}$ for $x = 0, 1, 2, 3, 4, 5, 6, 7$ \hspace{1cm} Yes
The Mean of a Probability Distribution

Find the mean of the value of $x$ when $x$ is the number of heads in 2 tosses of a coin.
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Find the mean of the value of $x$ when $x$ is the number of heads in 2 tosses of a coin. The possible values of $x$ are 0, 1, 1, and 2, so the mean can be found as follows.
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\[
\frac{0 + 1 + 1 + 2}{4}
\]
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Find the mean of the value of $x$ when $x$ is the number of heads in 2 tosses of a coin. The possible values of $x$ are 0, 1, 1, and 2, so the mean can be found as follows.

\[
\frac{0 + 1 + 1 + 2}{4} = \frac{0}{4} + \frac{1 + 1}{4} + \frac{2}{4}
\]
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$$= 0 \left( \frac{1}{4} \right) + 1 \left( \frac{2}{4} \right) + 2 \left( \frac{1}{4} \right)$$
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$$= 0 \left( \frac{1}{4} \right) + 1 \left( \frac{1}{4} \right) + 2 \left( \frac{1}{4} \right)$$

$$= 0P(0) + 1P(1) + 2P(2)$$
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\[
\frac{0 + 1 + 1 + 2}{4} = \frac{0}{4} + \frac{1 + 1}{4} + \frac{2}{4} = 0 \left(\frac{1}{4}\right) + 1 \left(\frac{2}{4}\right) + 2 \left(\frac{1}{4}\right) = 0P(0) + 1P(1) + 2P(2)
\]

**DEFINITION**

The mean or expected value of a discrete random variable is \( \mu = E(x) = \sum xp(x) \).
An insurance company sells a $10,000 one-year term life insurance policy for $290. The probability of death for a customer is .001. What can the insurance company expect to make on the sale of each policy?
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Let $x$ be the variable which measures the gain or loss on a policy. If the person lives then $x = 290$. If the person dies, then $x = 290 - 10000 = -9710$. 

The expected value is 

$$
290 \cdot P(290) - 9710 \cdot P(-9710) = (290)(.999) - (9710)(.001) = 280.
$$
The Mean of a Probability Distribution

An insurance company sells a $10,000 one-year term life insurance policy for $290. The probability of death for a customer is .001. What can the insurance company expect to make on the sale of each policy?

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The Variance of a Probability Distribution

Recall that the variance is the mean of the squares of the distances from the values to the mean of the data.
The Variance of a Probability Distribution

Recall that the variance is the mean of the squares of the distances from the values to the mean of the data. The mean of a random variable $x$ is

$$\sum x p(x).$$

**Definition**

The variance of a discrete random variable $x$ is

$$\sigma^2 = \sum (x - \mu)^2 p(x).$$

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The standard deviation of a discrete random variable is

$$\sigma = \sqrt{\sigma^2}.$$
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We can apply Chebyshev’s Rule and the Empirical Rule to these distributions.
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The Variance of a Probability Distribution

Let $x$ be a discrete random variable with probability distribution $p(x)$, mean $\mu$, and standard deviation $\sigma$. 

Chebyshev's Rule

- Empirical Rule
- $P(\mu - \sigma < x < \mu + \sigma) \geq \frac{1}{4} \approx 0.68$
- $P(\mu - 2\sigma < x < \mu + 2\sigma) \geq \frac{3}{4} \approx 0.95$
- $P(\mu - 3\sigma < x < \mu + 3\sigma) \geq \frac{8}{9} \approx 0.997$
The Variance of a Probability Distribution

Let $x$ be a discrete random variable with probability distribution $p(x)$, mean $\mu$, and standard deviation $\sigma$.

**Chebyshev’s Rule**

- $P(\mu - \sigma < x < \mu + \sigma) 
- P(\mu - 2\sigma < x < \mu + 2\sigma) 
- P(\mu - 3\sigma < x < \mu + 3\sigma)$

**Empirical Rule**
The Variance of a Probability Distribution

Let $x$ be a discrete random variable with probability distribution $p(x)$, mean $\mu$, and standard deviation $\sigma$.

Chebyshev’s Rule  Empirical Rule

\[ P(\mu - \sigma < x < \mu + \sigma) \geq 0 \]
\[ P(\mu - 2\sigma < x < \mu + 2\sigma) \approx \frac{3}{4} \]
\[ P(\mu - 3\sigma < x < \mu + 3\sigma) \approx \frac{8}{9} \]
The Variance of a Probability Distribution

Let $x$ be a discrete random variable with probability distribution $p(x)$, mean $\mu$, and standard deviation $\sigma$.

**Chebyshev’s Rule**  **Empirical Rule**

$P(\mu - \sigma < x < \mu + \sigma) \geq 0$

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Binomial Experiments

Characteristics of a Binomial Experiment

1. The experiment consists of $n$ identical trials.
2. There are only 2 possible outcomes: $S$ (for success) and $F$ (for failure).
3. The probability for $S$ is the same in each trial.
4. The trials are independent.
5. The binomial random variable $x$ is the number of $S$'s in the $n$ trials.

The probability of $S$ in a single trial is denoted $p$. Then the probability of $F$ is $q = 1 - p$. 
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Determine whether the following random variables are binomial.
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1. A committee is to be formed by randomly choosing 2 names from among 10 members, 6 of which are male and 4 are female. Let $x$ be the number of females chosen. The trials are not independent. This is not a binomial experiment.

2. A taste test is performed using 100 randomly chosen consumers. Each consumer can prefer either Brand A or Brand B. One tester does not influence another. Let $x$ be the number of consumers who prefer Brand A. There are 100 identical trials. There are exactly two possible outcomes. The probability of success is the same on each trial. The trials are independent. This is a binomial experiment.

3. Teacher evaluations: rate the professor on a scale of 0 to 5. Let $x$ be the number given. This is not a binomial experiment because $x$ can take more than one value.
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Binomial Probabilities

Consider a binomial experiment with 4 trials in which $p = .4$ and $q = .6$. 
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SSSS  SSSF  SSFF  SFFF  FFFF  
SSFS  SFSF  FSFF  
SFSS  SFFS  FFSF  
FSSS  FSFS  FFFS  
FFSS  FFSF  
FFSF
Binomial Probabilities

\[ P(x = 4) \]
Binomial Probabilities

\[ P(x = 4) = P(SSSS) \]
Binomial Probabilities

\[ P(x = 4) = P(SSSS) = P(S)P(S)P(S)P(S) \]
Binomial Probabilities

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\[ P(x = 3) \]
Binomial Probabilities

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Binomial Probabilities

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P(SSSF) = (.4)(.4)(.4)(.6)
Binomial Probabilities

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\[ P(SSSF) = (.4)(.4)(.4)(.6) = (.4)^3(.6) \]
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\[ P(SSSF) = (.4)(.4)(.4)(.6) = (.4)^3(.6)^1 \]
\[ P(SSFS) \]
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\[ P(x = 3) = P(SSSF) + P(SSFS) + P(SFSS) + P(FSSS) \]
\[ P(SSSF) = (0.4)(0.4)(0.4)(0.6) = (0.4)^3(0.6)^1 \]
\[ P(SSFS) = (0.4)(0.4)(0.6)(0.4) = (0.4)^3(0.6)^1 \]

In fact, all the combinations of 3 \( S \)'s and 1 \( F \) will have the same probability. So
\[ P(x = 3) = 4(0.4)^3(0.6)^1 \]
We can do the same thing with each of the types of combinations.
Binomial Probabilities

\[ P(x = 4) = P(SSSS) = P(S)P(S)P(S)P(S) = (.4)^4 \]
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We can do the same thing with each of the types of combinations.
Binomial Probabilities

\[ P(SSFF) \]

So \[ P(x = 2) \]

\[ 4(\cdot4)^2(\cdot6)^2 \]

Finally, \[ P(x = 0) = P(FFFF) = (\cdot6)^4 \]

To recap:

\[ P(x = 4) = (\cdot4)^4 \]
\[ P(x = 3) = 4(\cdot4)^3(\cdot6)^1 \]
\[ P(x = 2) = 6(\cdot4)^2(\cdot6)^2 \]
\[ P(x = 1) = 4(\cdot4)^1(\cdot6)^3 \]
\[ P(x = 0) = 1(\cdot4)^0(\cdot6)^4 \]
Binomial Probabilities

\[ P(SSFF) = (0.4)(0.4)(0.6)(0.6) \]
Binomial Probabilities

\[ P(SSFF) = (.4)(.4)(.6)(.6) = (.4)^2(.6)^2 \]
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\[ P(SFFF) \]
Binomial Probabilities

\[ P(SSFF) = (0.4)(0.4)(0.6)(0.6) = (0.4)^2(0.6)^2 \]
So \( P(x = 2) = 6(0.4)^2(0.6)^2. \)

\[ P(SFFF) = (0.4)(0.6)(0.6)(0.6) \]
Binomial Probabilities

\[ P(SSFF) = (.4)(.4)(.6)(.6) = (.4)^2(.6)^2 \]
So \( P(x = 2) = 6(.4)^2(.6)^2 \).

\[ P(SFFF) = (.4)(.6)(.6)(.6) = (.4)^1(.6)^3 \]
Binomial Probabilities

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Binomial Probabilities

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Binomial Probabilities

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So \[ P(x = 2) = 6(.4)^2(.6)^2. \]

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Binomial Probabilities

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**Binomial Probabilities**

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Binomial Probabilities

\[ P(SSFF) = (0.4)(0.4)(0.6)(0.6) = (0.4)^2(0.6)^2 \]
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So \( P(x = 1) = 4(0.4)(0.6)^3 \).

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To recap:
\[ P(x = 4) = (0.4)^4 \]
Binomial Probabilities

\[
P(\text{SSFF}) = (.4)(.4)(.6)(.6) = (.4)^2(.6)^2
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So \( P(x = 2) = 6(.4)^2(.6)^2 \).

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To recap:
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P(x = 4) = (.4)^4 = 1(.4)^4(.6)^0
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Combinations

Suppose you had $N$ books that you wished to arrange on a shelf.
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Combinations

Suppose you had \( N \) books that you wished to arrange on a shelf. We have a choice of \( N \) books for the first spot, \( N - 1 \) books for the second,
Combinations

Suppose you had $N$ books that you wished to arrange on a shelf. We have a choice of $N$ books for the first spot, $N - 1$ books for the second, $N - 2$ books for the third, and so on.

So there are $N(N-1)(N-2)\cdots1$ ways of arranging the books. This number is called $N$ factorial and is denoted $N!$.

**DEFINITION**

$N! = 1 \cdot 2 \cdot 3 \cdots N$ when $N > 0$ and $0! = 1$.

Now suppose that we only wanted to arrange $n$ out of the $N$ books. There are $N$ choices, then $N-1$, and so on, but we stop after $n$. So on our last choice we have $N-(n-1) = N-n+1$ books left. The number of ways of arranging $n$ out of $N$ books is $(N-n+1)\cdots N = 1\cdots N \cdots (N-n) = N!/(N-n)!$. 
Combinations

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Suppose you had $N$ books that you wished to arrange on a shelf. We have a choice of $N$ books for the first spot, $N - 1$ books for the second, $N - 2$ books for the third, and so on. So there are $N(N - 1)(N - 2)\cdots 1$ ways of arranging the books. This number is called $N$ factorial and is denoted $N!$.

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$N! = 1 \cdot 2 \cdot 3 \cdots N$ when $N > 0$ and $0! = 1$.

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There are a total of 24 (or $N! / (N-n)!$) arrangements divided into 4 groups.
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\[
\begin{array}{ccccccc}
123 & 213 & 231 & 321 & 312 & 132 \\
124 & 214 & 241 & 421 & 412 & 142 \\
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There are a total of 24 (or $N!/(N - n)!$) arrangements divided into 4 groups.
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There are a total of 24 (or \( N!/(N − n)! \)) arrangements divided into 4 groups. We get these four groups be dividing the total (24) by the number of ways that the three books we put into the bag can be arranged (\( n! \) or 6).
The Binomial Probability Distribution

Binomial Coefficients

\[
\binom{N}{n} = \frac{N!}{n!(N - n)!}
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The Binomial Probability Distribution

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Combinations Rule
If a sample of \( n \) elements is to be chosen from a set of \( N \) elements then the number of different samples that can be formed is \( \binom{N}{n} \).
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Combinations Rule

If a sample of \( n \) elements is to be chosen from a set of \( N \) elements then the number of different samples that can be formed is \( \binom{N}{n} \).

The Binomial Probability Distribution

If \( x \) is the random variable of a binomial experiment with \( n \) trials, \( P(S) = p \), and \( P(F) = q \), then

\[
p(x) = \binom{n}{x} p^x q^{n-x}
\]
Example

A study concludes that 25% of bottled water is just tap water. Suppose that a sample of 5 different bottled-water brands is randomly selected and that \( x \) is the number of bottles that contain tap water.

1. Is \( x \) a binomial variable? Yes.

2. Give a probability distribution for \( x \).

\[
p(x) = \binom{n}{x} p^x q^{n-x} = \binom{5}{x} (0.25)^x (0.75)^{5-x}
\]

3. Find \( p(x = 2) \).

\[
p(x = 2) = \binom{5}{2} (0.25)^2 (0.75)^{5-2} = (10)(0.2373) = 2.373
\]

4. Find \( p(x \leq 1) \).

\[
p(x \leq 1) = p(x = 0) + p(x = 1) = \binom{5}{0} (0.25)^0 (0.75)^{5-0} + \binom{5}{1} (0.25)^1 (0.75)^{5-1} = (5)(0.0625)(0.422) = 1.0625
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3. Find $P(x = 2)$.

$$P(x = 2) = \binom{5}{2} (0.25)^2 (0.75)^{5-2} = (10)(0.0625)(0.422) = 0.2637$$

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Using Table II.

Now suppose that the probability that a single bottle contains tap water is 20% and that 20 samples of bottled water are tested.

1. Find $p(x \leq 7)$.

2. Find $p(x \geq 8)$.

3. Find $p(x = 5)$.

4. Find $p(3 \leq x \leq 9)$.
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1. Find \( p(x \leq 7) \).
   \[ .968 \]

2. Find \( p(x \geq 8) \).
   \[ 1 - p(x \leq 7) = 1 - .968 = .032 \]

3. Find \( p(x = 5) \).
   \[ p(x = 5) = p(x \leq 5) - p(x \leq 4) = .804 - .630 = .174 \]

4. Find \( p(3 \leq x \leq 9) \).
   \[ p(3 \leq x \leq 9) = p(x \leq 9) - p(x < 3) = p(x \leq 9) - p(x \leq 2) = .997 - .206 = .791 \]
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The Mean and Standard Deviation of a Binomial Distribution

The Mean of a Binomial Random Variable

If a binomial experiment has $n$ trials and the probability of success on a single trial is $p$ then the mean or expected number of successes among the $n$ trials is $\mu = np$. 

If $n = 30$ and $p = 0.15$ then $\mu = 4.5$ and $\sigma = \sqrt{(30)(0.15)(0.85)} = \sqrt{3.825} = 1.96$. 
The Mean and Standard Deviation of a Binomial Distribution

The Mean of a Binomial Random Variable

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The Standard Deviation of a Binomial Random Variable

If a binomial experiment has \( n \) trials and the probability of success on a single trial is \( p \) then the variance of the number of successes is \( \sigma^2 = npq \) and the standard deviation is \( \sigma = \sqrt{npq} \).
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Continuous Random Variables

Recall that the probability of a discrete random variable can be associated with the area of a graph. In that case, we were graphing the probability of each possible value of the variable.
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Continuous Random Variables

What were the two properties that defined a probability function for a discrete random variable $x$?
Continuous Random Variables

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What were the two properties that defined a probability function for a discrete random variable $x$? $0 \leq p(x) \leq 1$ for all $x$ and $\sum p(x) = 1$. There are two similar properties that will define a probability density function $f(x)$ for a continuous variable $x$: $f(x) \geq 0$ for all $x$ and the area under the graph of $f$ is exactly 1.
Consider $f(x) = 2x$ for $0 \leq x \leq 1$. 
Consider \( f(x) = 2x \) for \( 0 \leq x \leq 1 \).

1. Find \( p(x \leq .7) \).
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p(.2 \leq x \leq .8) = p(x \leq .8) - p(x \leq .2) = .64 - .04 = .6
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The Normal Distribution

A very common distribution is the **normal distribution**. The graph of this distribution is the familiar symmetric bell-shaped curve.
The Normal Distribution

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**DEFINITION**

The *standard normal distribution* is a normal distribution with \( \mu = 0 \) and \( \sigma = 1 \). A random variable with a standard normal distribution is called a *standard normal random variable* and is denoted \( z \).
The Normal Distribution

It is difficult to find the areas under parts of this curve. Consequently, we use calculators or tables. In your book, Table III gives areas from 0 (the mean) to a designated positive value of $z$. This corresponds to a probability of the form $p(0 \leq z \leq z_0)$.

Here are some rules to keep in mind when finding probabilities with Table III:

1. Table III only gives probabilities of the form $p(0 \leq z \leq z_0)$.
2. If $z_0 < 0$ then $p(z_0 \leq z \leq 0) = p(0 \leq z \leq -z_0)$.
3. $p(z_0 \leq z)$.
4. If $z_0 < 0 < z_1$ then $p(z_0 \leq z \leq z_1) = p(0 \leq z \leq z_1) + p(0 \leq z \leq z_0)$.
5. If $z_0 < 0$ then $p(z \leq z_0) = 0.5 - p(z_0 \leq z \leq 0) = 0.5 - p(0 \leq z \leq -z_0)$.
6. If $0 < z_0 < z_1$ then $p(z_0 \leq z \leq z_1) = p(0 \leq z \leq z_1) - p(0 \leq z \leq z_0)$.
7. It always helps to draw a picture.
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If $x_1$ and $x_2$ are values of a nonstandard normal random variable with corresponding $z$-scores $z_1$ and $z_2$ then $p(x_1 \leq x \leq x_2) = p(z_1 \leq z \leq z_2)$. 
Examples of the Nonstandard Normal Distribution

A company makes yardsticks and studies show that the mean length of the yardsticks is 36 inches with a standard deviation of .1 inches.

\[ z(36) = 0 \] and \[ z(36.25) = 2.5 \] so \[ p(36 \leq x \leq 36.25) = p(0 \leq z \leq 2.5) = 0.4938. \]

\[ z(35.95) = -0.5 \] and \[ z(36.05) = 0.5 \] so \[ p(35.95 \leq x \leq 36.05) = p(-0.5 \leq z \leq 0.5) = 0.383. \]

\[ z(36.15) = 1.5 \] so \[ p(36.15 < x) = p(1.5 < z) = p(0 \leq z \leq 1.5) = 0.5 - p(0 \leq z \leq 1.5) = 0.5 - 0.4332 = 0.0668. \]
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Find the probability that a randomly selected yardstick has length between 36.01 and 36.1 inches.

$z(36.01) = .1$ and $z(36.1) = 1$ so

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5 If 10,000 yardsticks are made, how many would we expect to be .125 inches too long?
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   \[ z(36.125) = 1.25 \text{ so } p(36.125 \leq x) = p(1.25 \leq z) = \]
   \[ .5 - p(0 \leq z \leq 1.25) = .5 - .3849 = .1151. \text{ Then } 11.51\% \text{ of } 10,000 \text{ is } 1151. \]
Examples of the Nonstandard Normal Distribution

6 Find the 30th percentile of a normal distribution whose mean is 50 and whose standard deviation is 17.

This time we want to know the value of $z$ which makes $P(z \leq z_{0.3}) = 0.3$. Since $P(z \leq z_{0.3}) < 0.5$, we know that $z_{0.3} < 0$. Now $P(0 \leq z \leq z_{0.3}) = 0.5 - P(z < z_{0.3}) = 0.5 - 0.2 = 0.3$. The closest we can get to 0.3 is 0.1985, whose corresponding $z$-score is $-0.52$. Thus $z_{0.3} = -0.52$. The value of $x$ is then $z_{0.3} \sigma + \mu = (-0.52)(17) + 50 = 41.2$.

7 Find the 60th percentile of a normal distribution whose mean is 50 and whose standard deviation is 17.

We must find $z_{0.6}$, the $z$-score with the property that $P(z \leq z_{0.6}) = 0.6$. Since this probability is greater than 0.5, we know that $0 < z_{0.6}$. Now $P(0 \leq z \leq z_{0.6}) = 0.6 - 0.5 = 0.1$ and the closest that we can get to 0.1 in Table III is 0.0987. So $z_{0.6} = 0.25$ and the corresponding value of $x$ is $(0.25)(17) + 50 = 54.25$. 
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean:</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Variance:</td>
<td>$\sigma^2$</td>
</tr>
<tr>
<td>Standard Deviation:</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Binomial Proportion:</td>
<td>$p$</td>
</tr>
</tbody>
</table>
DEFINITION
The sampling distribution of a sample statistic calculated from a sample of size n is the probability distribution of the statistic for all possible samples of size n.
DEFINITION

The *sampling distribution* of a sample statistic calculated from a sample of size $n$ is the probability distribution of the statistic for all possible samples of size $n$.

We will create several sampling distributions based on a population with two members: 0 and 1. The probability of randomly choosing 0 from the population is $1/3$ and the probability of randomly choosing 1 is $2/3$. We will first obtain the mean and standard deviation of the population, then for samples of size 2, then for samples of size 3.
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The Population Distribution
The Population Distribution

<table>
<thead>
<tr>
<th>$x$</th>
<th>$p(x)$</th>
<th>$xp(x)$</th>
<th>$(x - \mu)^2 p(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/3</td>
<td>0</td>
<td>4/27</td>
</tr>
<tr>
<td>1</td>
<td>2/3</td>
<td>2/3</td>
<td>2/27</td>
</tr>
<tr>
<td>sum</td>
<td>1</td>
<td>2/3</td>
<td>2/9</td>
</tr>
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<td>sum</td>
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<td>2/3</td>
<td>2/9</td>
</tr>
</tbody>
</table>

$\mu = \frac{2}{3}$

$\sigma = \frac{\sqrt{2}}{3}$
The Distribution for Samples of Size 2
The Distribution for Samples of Size 2

<table>
<thead>
<tr>
<th>$\bar{x}$</th>
<th>$p(\bar{x})$</th>
<th>$\bar{x} p(\bar{x})$</th>
<th>$(\bar{x} - \mu_{\bar{x}})^2 p(\bar{x})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/9</td>
<td>0</td>
<td>4/81</td>
</tr>
<tr>
<td>1/2</td>
<td>4/9</td>
<td>2/9</td>
<td>1/81</td>
</tr>
<tr>
<td>1</td>
<td>4/9</td>
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<td>4/81</td>
</tr>
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</tr>
<tr>
<td>sum</td>
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<td>1/9</td>
</tr>
</tbody>
</table>

$\mu_{\bar{x}} = \frac{2}{3} = \mu$

$\sigma_{\bar{x}} = \frac{1}{3} = \frac{\sqrt{2}}{3} \cdot \frac{1}{\sqrt{2}} = \frac{\sigma}{\sqrt{2}}$
The Distribution for Samples of Size 3
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<table>
<thead>
<tr>
<th>$\bar{x}$</th>
<th>$p(\bar{x})$</th>
<th>$\bar{x}p(\bar{x})$</th>
<th>$(\bar{x} - \mu_{\bar{x}})^2 p(\bar{x})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/27</td>
<td>0</td>
<td>4/243</td>
</tr>
<tr>
<td>1/3</td>
<td>6/27</td>
<td>6/81</td>
<td>6/243</td>
</tr>
<tr>
<td>2/3</td>
<td>12/27</td>
<td>24/81</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>8/27</td>
<td>24/81</td>
<td>8/243</td>
</tr>
<tr>
<td>sum</td>
<td>1</td>
<td>2/3</td>
<td>2/27</td>
</tr>
</tbody>
</table>
The Distribution for Samples of Size 3

<table>
<thead>
<tr>
<th>( \bar{x} )</th>
<th>( p(\bar{x}) )</th>
<th>( \bar{x}p(\bar{x}) )</th>
<th>( (\bar{x} - \mu_{\bar{x}})^2 p(\bar{x}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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\]
The Histogram of the Population

Here is the histogram of the original population.
The Histogram of the Population

Here is the histogram of the original population.
The Histogram of the Samples of Size 2
The Histogram of the Samples of Size 3

![Histogram](chart.png)
The Histogram of the Samples of Size 10
The Central Limit Theorem

Consider the probability distribution of the means, $\bar{x}$, of all possible samples of size $n$ drawn randomly from a population with mean $\mu$ and standard deviation $\sigma$. 

1. The mean of the distribution is $\mu_{\bar{x}} = \mu$.
2. The standard deviation of the distribution is $\sigma_{\bar{x}} = \sigma \sqrt{n}$.
3. The distribution is approximately normal when $n$ is sufficiently large.
4. The larger $n$ is, the closer the distribution becomes to a normal distribution.
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Consider the probability distribution of the means, \( \bar{x} \), of all possible samples of size \( n \) drawn randomly from a population with mean \( \mu \) and standard deviation \( \sigma \).

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If we know that the population is normal, we can be more specific.
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THEOREM

The probability distribution of the means of all possible samples of size n drawn randomly from a normally distributed population will be normally distributed.
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We can see from the histogram of the size 3 example that the distribution is becoming more normal.
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The population from which that samples are chosen has mean 20 and standard deviation 16. Use $n = 64$.
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Examples of The Central Limit Theorem

Now use $n = 400$. 

1. What change does this make in the shape of the distribution of $x$? It is still normal, but is now narrower and more concentrated about the mean.

2. Find $\mu_x$ and $\sigma_x$.

$$\mu_x = 20 \text{ and } \sigma_x = 0.8.$$ 

3. Find the probability that a randomly chosen sample of size 400 has a mean larger than 22.

$$P(X > 22) = 0.0062.$$
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Estimators

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The sample mean $\bar{x}$ is an example of a point estimator.

This does not imply that all means of large samples are the same as the mean of the population. They could be very different.
How confident can we be of our estimation?

Recall that the distribution for the sample means is approximately normal. Since the mean of the distribution of the sample means is the same as the mean of the population, \( \bar{x} \) is an example of an unbiased estimator of \( \mu \).

**DEFINITION**

A sample statistic is said to be an unbiased estimator of a population parameter if the mean of the distribution of the statistic has the same value as the parameter. A statistic that is not unbiased is called biased.

In order to be a good estimator of a parameter a statistic should be unbiased and have low variability (small standard deviation).
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Estimators

Is the statistic given by the blue line a good estimator of the parameter given by the red line?
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$$\mu = \bar{x} \pm 2\sigma_{\bar{x}} = \bar{x} \pm 2\frac{\sigma}{\sqrt{n}}$$
Estimators

One problem with this formula is that we don’t know $\sigma$. Since the sample is large, the sample standard deviation should be close in value to $\sigma$, so we will use $s$ in place of $\sigma$. Then $\mu = \bar{x} \pm \frac{2s}{\sqrt{n}}$, or $\bar{x} - \frac{2s}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{2s}{\sqrt{n}}$. 

The Empirical Rule tells us that 95% of the samples we can select randomly will have a sample mean that is within 2 standard deviations of $\mu$. That means that we are 95% confident that $\mu$ is within the interval that we gave above.

If we randomly select a sample of size 64 which has a sample mean of 23 and a standard deviation of 16, then we are 95% sure that the population mean is $23 \pm \frac{2 \cdot 16}{\sqrt{64}} = 23 \pm 5$. We are 95% sure that $\mu$ is between 22.5 and 23.5.
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Estimators

One problem with this formula is that we don’t know \( \sigma \). Since the sample is large, the sample standard deviation should be close in value to \( \sigma \), so we will use \( s \) in place of \( \sigma \). Then \( \mu = \bar{x} \pm \frac{2s}{\sqrt{n}} \), or \( \bar{x} - \frac{(2s)}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{(2s)}{\sqrt{n}} \).

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Confidence Intervals

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A *confidence interval* (or *interval estimate*) of a population parameter is a formula that tells how to use sample data to calculate an interval that estimates the parameter.
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A *confidence interval* (or *interval estimate*) of a population parameter is a formula that tells how to use sample data to calculate an interval that estimates the parameter.

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The *confidence coefficient* of an confidence interval for a population parameter is the probability that the interval encloses the parameter, that is, the relative frequency with which the confidence interval encloses the parameter when the estimator is used a large number of times. The *confidence level* is the confidence coefficient expressed as a percentage.
Other Confidence Levels

What if we want a confidence level other than 95%?

DEFINITION

The value of $z_{\alpha}$ is the value of the standard normal random variable $z$ such that the area of the region to the right of $z$ is $\alpha$. That is, $P(z > z_{\alpha}) = \alpha$.

$100(1 - \alpha) = \alpha \alpha/2$

90% .10 .05 1.645

95% .05 .025 1.96

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<tr>
<th></th>
<th>100(1 - $\alpha$)</th>
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<th>$\alpha/2$</th>
<th>$z_{\alpha/2}$</th>
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<tbody>
<tr>
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Large-Sample 100(1 - \(\alpha\))% Confidence Interval for \(\mu\)

The large sample 100(1 - \(\alpha\))% confidence interval for \(\mu\) is

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\bar{x} \pm z_{\alpha/2} \sigma_{\bar{x}} = \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \approx \bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}
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Here \(n\) is at least 30.
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We can use SPSS to find confidence intervals based on samples.
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When large samples are not available and we must rely on a small sample, we quickly run into two problems.
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Solutions to the Two Problems

Here is how we solve these two problems.

1. Use only populations that are nearly normally distributed.

We know that if the population is normally distributed then the sample means will be normally distributed.

2. Rather than $z$, use a different random variable $t$ which depends on $x$ and $s$ rather than $x$ and $\sigma$.

Recall that the sample variance is $s^2 = \frac{\sum (x - x)^2}{n-1}$ so it depends on the sample mean and the size of the sample. Thus the new variable $t$ will depend on both of those things. The value $n-1$ is called the degrees of freedom of $t$ and is denoted $df$. 
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The shape of the distribution of \( t \) is basically that of the standard normal distribution:

Some of these values can be found in Table IV on page 568.
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1. Find $t_{1.1}$ when $df = 27$. 

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We can still use SPSS to find confidence intervals. This example uses LM5_32.sav.

1. Find the 80% confidence interval for the population mean.
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Conditions Required for a Valid Estimate of $p$

1. A random sample is selected from the target population.
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Properties of the Sampling Distribution of \( \hat{p} \)

1. The mean of the sampling distribution of \( \hat{p} \) is \( p \).
2. The standard deviation of the sampling distribution of \( \hat{p} \) is \( \sigma_{\hat{p}} = \sqrt{pq/n} \).
3. For large samples the sampling distribution of \( \hat{p} \) is approximately normal.

Large-Sample 100(1 - \( \alpha \))\% Confidence Interval for \( p \)

The large-sample confidence interval for \( p \) is

\[ \hat{p} \pm z_{\alpha/2} \sigma_{\hat{p}} = \hat{p} \pm z_{\alpha/2} \sqrt{pq/n} \approx \hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}\hat{q}/n} \]

where \( \hat{p} = x/n \) and \( z_{\alpha/2} \) is based on a sample of size \( n \).
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2. SPSS will not find confidence intervals for populations proportions because the standard deviation is calculated in a different way for the proportion than for the mean.
Sample Size for Estimating $\mu$

We wish to determine the size of the sample that we must generate in order to calculate the $100(1 - \alpha)\%$ confidence interval for $\mu$. 

\[
|\mu - x| \leq z_{\alpha/2} \sigma \sqrt{n}
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The value $z_{\alpha/2} \sigma \sqrt{n}$ measures that difference between the actually mean of the population and our approximation based on a sample. This is called the sampling error and is denoted SE.
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Sample Size for the 100(1 - $\alpha$)% Confidence Interval for $\mu$

*In order to estimate $\mu$ with sampling error $SE$ and with 100(1 - $\alpha$)% confidence, the required sample size is at least*

\[
n = \frac{(z_{\alpha/2})^2\sigma^2}{SE^2}
\]
Sample Size for Estimating $\mu$

If $\sigma$ is unknown, there are two options that we can use to approximate it.

1. If $s$ is known for a previous sample, we can use $s$ to approximate $\sigma$.
2. If a range $R$ for the values of $x$ is given then we can use $R/4$ to approximate $\sigma$.

Always round the size that you obtain for the sample up.

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Always round the size that you obtain for the sample up.
If $\sigma$ is unknown, there are two options that we can use to approximate it.

1. If $s$ is known for a previous sample, we can use $s$ to approximate $\sigma$.
2. If a range $R$ for the values of $\bar{x}$ is given then we can use $R/4$ to approximate $\sigma$.

Always round the size that you obtain for the sample up. Since these calculations are based on a large-sample approximation, the sample that one uses must always be at least 30.
Sample Size for Estimating $\mu$

1. Find the sample size needed to estimate the mean of a population to within .25 with a confidence of 99%. The standard deviation of the population is known to be 1.2.
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2. Repeat the above example with SE = .5, then with a confidence of 80%.
Sample Size for Estimating $\mu$

1. Find the sample size needed to estimate the mean of a population to within 0.25 with a confidence of 99%. The standard deviation of the population is known to be 1.2.

2. Repeat the above example with SE = 0.5, then with a confidence of 80%.

3. An FDA study indicates that the average cup of coffee contains 115 mg of caffeine. The standard deviation of the sample used in this study was 30. How many cups of coffee need to be tested in order to estimate the average amount of caffeine in a cup of coffee to within 5 mg with a confidence of 95%?
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4. A study finds that the mean weight of weight of cereal in a box is 24 oz, with values varying between 21 oz and 25 oz. How many boxes must be sampled to estimate the mean weight of cereal in a box to within .5 oz with a certainty of 99%?
Sample Size for Estimating $p$

We saw earlier that $SE = z_{\alpha/2} \sigma_x$.


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Sample Size for the 100$(1 - \alpha)$% Confidence Interval for $p$

The sample size needed to estimate a binomial probability $p$ with sampling error $SE$ and with 100$(1 - \alpha)$% confidence is at least

$$n = \frac{(z_{\alpha/2})^2pq}{SE^2}.$$
Sample Size for Estimating $p$

Obviously, $p$ is an unknown quantity, and therefore $q$ is as well. We have two options for approximating $p$.
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1. If $\hat{p}$ is known from a previous sample, we can use it to approximate $p$. 
Sample Size for Estimating $p$

Obviously, $p$ is an unknown quantity, and therefore $q$ is as well. We have two options for approximating $p$.

1. If $\hat{p}$ is known from a previous sample, we can use it to approximate $p$.

2. The maximum value of $pq$ is .25, so we can use it as an approximation for $pq$ and know that the resulting sample size is at least as big as the one that we need.
Sample Size for Estimating $p$

Obviously, $p$ is an unknown quantity, and therefore $q$ is as well. We have two options for approximating $p$.

1. If $\hat{p}$ is known from a previous sample, we can use it to approximate $p$.
2. The maximum value of $pq$ is .25, so we can use it as an approximation for $pq$ and know that the resulting sample size is at least as big as the one that we need.

Always round $n$ up.
A manufacturer wishes to estimate the proportion of its product that is defective. This estimate should be within .01 of the true value and have a confidence level of 90%. What size sample is needed?
Sample Size for Estimating $p$

1. A manufacturer wishes to estimate the proportion of its product that is defective. This estimate should be within .01 of the true value and have a confidence level of 90%. what size sample is needed?

2. That size sample is too big. What can we do to reduce the size of the sample?
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That size sample is too big. What can we do to reduce the size of the sample? Find the sample size needed if we estimate $p$ to be .1.
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1. A manufacturer wishes to estimate the proportion of its product that is defective. This estimate should be within .01 of the true value and have a confidence level of 90%. What size sample is needed?

2. That size sample is too big. What can we do to reduce the size of the sample? Find the sample size needed if we estimate $p$ to be .1. Find the sample size needed if we keep the estimate of $pq$ at .25 but reduce the confidence to 80%.
Basics of Hypothesis Testing

Suppose that codes require that the pipes installed in a city have an average breaking point that is more than 2400 pounds per foot. When a company wishes to sell pipes to the city they will, of course, claim that their pipes meet this requirement.
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To be safe, the city will want to assume that the pipe fails to meet the requirement unless there is significant evidence to the contrary. That is, if \( \mu \) is the averaging breaking point then the city will assume that \( \mu \leq 2400 \).
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The null hypothesis, denoted $H_0$, is a statement which should be considered true unless there is significant evidence to the contrary. It’s statement always contains an equality ($=, \leq, \text{or } \geq$).
Another hypothesis we will consider is the alternative hypothesis. That is the opposite statement of the null hypothesis. It is the statement that will be accepted only if there is convincing evidence of its truth. It is denoted by $H_a$. 
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What should the city do to test this hypothesis? We will use the fact that only unusual samples will be far (in terms of the standard deviation) from the mean of the sample distribution.
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In our example of the city pipes, the alternative hypothesis is $\mu > 2400$.

What should the city do to test this hypothesis? We will use the fact that only unusual samples will be far (in terms of the standard deviation) from the mean of the sample distribution. The sample that we use to test the hypothesis is called the test statistic.
Suppose that the city tests a sample of 49 pipes and finds that the mean breaking point of the sample is 2450.
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Suppose that the city tests a sample of 49 pipes and finds that the mean breaking point of the sample is 2450. To see how we should interpret this result let’s see where it sits within the sample distribution. Assume that the standard deviation for all pipe mean breaking points is 238.

\[
    z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}
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From Table III we see that only 7% of all samples of size 49 will have a mean breaking point of 2450 or larger if the true mean of all pipes is 2400.
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Table 6.2 on page 309 gives rejection regions for common values of $\alpha$. When large samples ($n \geq 30$) are used, the sample standard deviation can be used to approximate the population standard deviation.
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Large-Sample Hypothesis Tests

1. Test the claim that $\mu \leq 40$ given a sample of size 150 which has a mean of 41.6 and a standard deviation of 9. Use $\alpha = .01$. 
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1. Test the claim that \( \mu \leq 40 \) given a sample of size 150 which has a mean of 41.6 and a standard deviation of 9. Use \( \alpha = .01 \).

2. Test the claim that \( \mu = 65 \) given a sample of size 50 with a mean of 66.1 and a standard deviation of 4. Use \( \alpha = .05 \).
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2. Test the claim that $\mu = 65$ given a sample of size 50 with a mean of 66.1 and a standard deviation of 4. Use $\alpha = .05$.

3. Test the claim that the average student commutes more than 9 miles to school, given a sample of 50 students with a mean of 10.22 and a standard deviation of 5. Use $\alpha = .05$. 
Large-Sample Hypothesis Tests

In the case of a two-tailed test, we are simply constructing the $100(1 - \alpha)\%$ confidence interval about the hypothesized value of $\mu$. If the test statistic (the $z$-score of the sample mean) falls within that interval, then we fail to reject $H_0$. If it falls without the interval, then we reject $H_0$.

If we construct this interval about the test statistic rather than the hypothesized mean, then we can use SPSS to construct the interval and check to see whether $\mu$ lies within it. If $\mu$ is in the interval then we fail to reject $H_0$ and if $\mu$ is not in the interval then we reject $H_0$.

Let’s try some problems, using SPSS to construct the confidence interval about the test statistic

Note that when we do one-tailed test, we will need to double the value of $\alpha$. 
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Let’s try some problems, using SPSS to construct the confidence interval about the test statistic. Note that when we do one-tailed test, we will need to double the value of $\alpha$. 
Before we do the example, we need to see how to come to the proper conclusion about the hypothesis from the kind of confidence interval that SPSS gives us. Let the confidence interval be $x_0 \leq \bar{x} \leq x_1$ and let $\mu_0$ be the hypothesized population mean.

1. If $H_a$ is $\mu \neq \mu_0$ then we reject $H_0$ if and only if $\mu_0 < x_0$ or $x_1 < \mu_0$.

2. If $H_a$ is $\mu < \mu_0$ then we reject $H_0$ if and only if $x_1 < \mu_0$.

3. If $H_a$ is $\mu > \mu_0$ then we reject $H_0$ if and only if $\mu_0 < x_0$. 

Use BONES.sav for the following examples.

1. Test the claim that $\mu \neq 8$ using $\alpha = 0.01$.

2. Test the claim that $\mu \leq 9$ using $\alpha = 0.05$.

3. Test the claim that $\mu < 9.5$ using $\alpha = 0.05$. 
Before we do the example, we need to see how to come to the proper conclusion about the hypothesis from the kind of confidence interval that SPSS gives us. Let the confidence interval by \( x_0 \leq \bar{x} \leq x_1 \) and let \( \mu_0 \) be the hypothesized population mean.

1. If \( H_a \) is \( \mu \neq \mu_0 \) then we reject \( H_0 \) if and only if \( \mu_0 < x_0 \) or \( x_1 < \mu_0 \).
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Use BONES.sav for the following examples.

1. Test the claim that \( \mu \neq 8.5 \) using \( \alpha = .01 \).
Large-Sample Hypothesis Tests

Before we do the example, we need to see how to come to the proper conclusion about the hypothesis from the kind of confidence interval that SPSS gives us. Let the confidence interval by $x_0 \leq \bar{x} \leq x_1$ and let $\mu_0$ be the hypothesized population mean.

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Use BONES.sav for the following examples.

1. Test the claim that $\mu \neq 8.5$ using $\alpha = .01$.

2. Test the claim that $\mu \leq 9$ using $\alpha = .05$. 
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Before we do the example, we need to see how to come to the proper conclusion about the hypothesis from the kind of confidence interval that SPSS gives us. Let the confidence interval by $x_0 \leq \bar{x} \leq x_1$ and let $\mu_0$ be the hypothesized population mean.

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2. Test the claim that $\mu \leq 9$ using $\alpha = .05$. 
Large-Sample Hypothesis Tests

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When we are relying on a small sample to test a claim about a population mean, the distribution of the samples means may no longer be normal.

\[ t = \frac{x - \mu}{s/\sqrt{n}}. \]

We will need to start with a population that is about normal. SPSS always uses the \( t \)-distribution, so it can be used to do small-sample hypothesis tests.

Use the sample \{12, 14, 15, 16, 16, 17, 19, 19, 20, 22\} to test the claim that \( \mu < 20 \). The mean of the sample is 17.6 and the standard deviation is 3.0. Use \( \alpha = 0.05 \). Check the answer with SPSS.
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When we are relying on a small sample to test a claim about a population mean, the distribution of the samples means may no longer be normal. For small samples we use the $t$-distribution. The formula for the $t$-distribution is $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$. We will need to start with a population that is about normal. SPSS always uses the $t$-distribution, so it can be used to do small-sample hypothesis tests. Use the sample \{12, 14, 15, 16, 16, 17, 19, 19, 20, 22\} to test the claim that $\mu < 20$. The mean of the sample is 17.6 and the standard deviation is 3.0. Use $\alpha = 0.05$. Check the answer with SPSS.
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Observed Significance Levels: $p$-values

In the hypothesis tests we have seen so far the value of $\alpha$ which determines the rejection region is chosen before the test is done.
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**DEFINITION**

The observed significance level, or \( p \)-value, for a statistical test is the probability (assuming \( H_0 \) is true) of observing a value of the test statistic that is at least as contrary to the null hypothesis as the one computed from the sample data.
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Observed Significance Levels: $p$-values

The $p$-value is the area of the region “beyond” the test statistic. For a one-tailed test, that is the area to the right or left of the test statistic, depending on $H_a$. For a two-tailed test, that is the combined area beyond both the test statistic and its negative.
Observed Significance Levels: \( p \)-values

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If an \( \alpha \) is chosen, then when the \( p \)-value (\( p \)) is less than \( \alpha \), the test statistic would be in the rejection region.
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If an \( \alpha \) is chosen, then when the \( p \)-value \((p)\) is less than \( \alpha \), the test statistic would be in the rejection region. If \( p > \alpha \) then the test statistic is not in the rejection region.

Calculating the \( p \)-Value for a Hypothesis Test

1. Determine the value of the test statistic \( z \).
2. If \( H_a \): \( \mu < \mu_0 \) then the \( p \)-value is the area of the tail to the left of \( z \).
3. If \( H_a \): \( \mu > \mu_0 \) then the \( p \)-value is the area of the tail to the right of \( z \).
4. If \( H_a \): \( \mu \neq \mu_0 \) then the \( p \)-value is twice the area of the tail to the right of \(|z|\).

If the test sample is small then use \( t \) rather than \( z \).
Observed Significance Levels: \( p \)-values

The \( p \)-value is the area of the region “beyond” the test statistic. For a one-tailed test, that is the area to the right or left of the test statistic, depending on \( H_a \). For a two-tailed test, that is the combined area beyond both the test statistic and its negative.

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Calculating the \( p \)-Value for a Hypothesis Test

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4. If \( H_a : \mu \neq \mu_0 \) then the \( p \) value is twice the area of the tail to the right of \( |z| \).
After the $p$-value is found, then $\alpha$ must be determined. If $p < \alpha$ then we reject $H_0$. If $p \geq \alpha$ then we fail to reject $H_0$. Use HOSPLOS.sav and $p$-values in the following examples.

1. Test the hypothesis that the mean of the population is less than 5.
2. Test the hypothesis that the mean of the population is greater than 4.
3. Test the hypothesis that the mean of the population is equal to 4.
Observed Significance Levels: $p$-values

After the $p$-value is found, then $\alpha$ must be determined. If $p < \alpha$ then we reject $H_0$. If $p \geq \alpha$ then we fail to reject $H_0$.

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1. Test the hypothesis that the mean of the population is less than 5.

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Use HOSPLOS.sav and $p$-values in the following examples.

1. Test the hypothesis that the mean of the population is less than 5.
2. Test the hypothesis that the mean of the population is greater than 4.
3. Test the hypothesis that the mean of the population is equal to 4.5.
Large-Sample Test of Hypothesis about a Population Proportion

When we test a hypothesis about a population proportion, we use the same approach as in our previous tests. The test statistic is calculated differently.

\[
 z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}
\]

Here \( p_0 \) is the hypothesized population proportion and \( \sigma_{\hat{p}} = \sqrt{\frac{p_0 q_0}{n}} \). In order for the test to be valid, we must meet the following two conditions.

1. A random sample is selected from a binomial population.
2. The sample size is large (\( np_0 \geq 15 \) and \( nq_0 \geq 15 \)).
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In a study of what “Made in the USA” means to consumers, 64 of 106 randomly chosen shoppers believed that “Made in the USA” means that 100% of labor and materials are from the USA.
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2. Test the claim that $p < .7$.
3. Test the claim that $p \leq .5$. 
A university developed a mathematics placement test for incoming students. The score needed to get into a certain class is 75%. A trial test was given to 38 students and the scores are given in scores.sav. Use this sample to test the claims given below and use several values of $\alpha$ for each test.
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A university developed a mathematics placement test for incoming students. The score needed to get into a certain class is 75%. A trial test was given to 38 students and the scores are given in scores.sav. Use this sample to test the claims given below and use several values of $\alpha$ for each test.

1. At least 60% of the incoming students will pass the test.
2. Less than 50% of the incoming students will pass the test.
3. More than 40% of the incoming students will pass the test.
Bivariate Relationships and their Graphs

Studies are often conducted to see if there is a connection between two variables.

How does the dosage of a certain medicine affect blood pressure? Is there a connection between religious belief and longevity? Is there a connection between the amount of sleep a student gets and GPA?

Connection does not mean causation!

Bivariate data is given in pairs, and we can graph the pairs in a two-dimensional coordinate system, with each axis representing one of the variables. Such a graph is called a scattergram or scatterplot.
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Bivariate data is given in pairs, and we can graph the pairs in a two-dimensional coordinate system, with each axis representing one of the variables. Such a graph is called a scattergram or scatterplot.
An Example of a Scattergram

Use SPSS to draw a scattergram of ALWINS.sav. This data pairs batting average with wins in 2007 American League teams.
Probabilistic Models

We can see from the scattergram that the batting average by itself cannot completely predict the number of wins.
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**DEFINITION**

*If there is a rule that can accurately predict the value of $y$ from the value of $x$, then the rule forms a deterministic model of the relation between $x$ and $y$. If there is a rule which can predict the value of $y$ from the value of $x$, except for some random error, then the rule forms a probabilistic model of the relation between $x$ and $y.*
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There is always a deterministic component to a probabilistic model; a function which approximates the value of y based on the value of x.
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There is always a deterministic component to a probabilistic model; a function which approximates the value of y based on the value of x. We will use the simplest possible deterministic function: a linear function.
Straight-Line Models

Linear Probablistic Model

\[ y = \beta_0 + \beta_1 x + \epsilon \]

where

- \( y \) = dependent or response variable
- \( x \) = independent or predictor variable
- \( \beta_0 \) = y-intercept of the line
- \( \beta_1 \) = slope of the line
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If $\beta_1 > 0$ then the line goes up to the right.
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The equation $y = \beta_0 + \beta_1 x$ is called the deterministic component of the linear model.

If $\beta_1 > 0$ then the line goes up to the right. If $\beta_1 < 0$ then the line goes down to the right.
The Coefficient of Correlation

How can we tell whether it is even possible to use a linear model on some bivariate data?
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But we need something better than a guess based on a graph. This is the purpose of the coefficient of correlation.

DEFINITION

The coefficient of correlation, \( r \), is a measure of the strength of the linear relationship between to variables.
The Coefficient of Correlation

Interpreting $r$

If $r$ is close to $-1$ then the variables have a strong negative linear correlation.

If $r$ is close to 1 then the variables have a strong positive linear correlation.

If $r$ is close to 0 then the variables have little or no linear correlation.

We can use SPSS to find the value of this coefficient.
The Coefficient of Correlation

Interpreting $r$

1. $-1 \leq r \leq 1$
The Coefficient of Correlation

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2. If $r$ is close to $-1$ then the variables have a strong negative linear correlation.
3. If $r$ is close to 1 then the variables have a strong positive linear correlation.
4. If $r$ is close to 0 then the variables have little or no linear correlation.

We can use SPSS to find the value of this coefficient.
Least Squares

Recall that the deterministic component of the linear probabilistic model is $y = \beta_0 + \beta_1 x$. How can we find $\beta_0$ and $\beta_1$? We will have to approximate them. What can we use to determine how good the approximation is? The errors are the vertical distances between the observed and predicted values. If an observed value is above the predicted value, the error is positive. If an observed value is below the predicted value, the error is negative. We want to choose approximations $\hat{\beta}_0$ and $\hat{\beta}_1$ so that the following conditions are satisfied.

1. The sum of the errors equals 0.
2. The sum of the squares of the errors is as small as possible.
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The values of $\hat{\beta}_0$ and $\hat{\beta}_1$ which do this are called the least squares estimates of the population parameters $\beta_0$ and $\beta_1$. The line $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ is called the least squares line. We can use SPSS and the regression function to find the values of $\hat{\beta}_0$ and $\hat{\beta}_1$. The least squares line can be used to make predictions. But the independent variable must be within the range of observed values.
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