STUDY GUIDE FOR MTH 218 TEST 4

Test 4 is scheduled for Tuesday, April 22. It will cover parts of the following sections from your book.

16.9
17.1–7

Be familiar with the following rules and definitions.

1. Jacobian
2. vector field
3. conservative vector field
4. potential
5. curl
6. divergence
7. line integral
8. independent of path
9. positively oriented curve
10. parametric surface

Be familiar with the following rules and theorems.

1. If \( x \) and \( y \) are functions of \( u \) and \( v \) then the Jacobian of \( x \) and \( y \) with respect to \( u \) and \( v \) is \( \frac{\partial (x, y)}{\partial (u, v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} \).

   You should also know the 3-dimensional version of the Jacobian.

2. (Change of variables) \( \iint_R f(x, y) \, dy \, dx = \iint_S f(x(u, v), y(u, v)) \left| \frac{\partial (x, y)}{\partial (u, v)} \right| \, dv \, du \) and \( \iint_R f(x, y, z) \, dy \, dz = \iint_S f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \frac{\partial (x, y, z)}{\partial (u, v, w)} \right| \, dw \, dv \, du \)

Be familiar with the following rules and theorems.

1. **Test for a Conservative Vector Field** Let \( \mathbf{F} = \langle M, N \rangle \) be a vector field on an open simply connected region \( D \) in \( \mathbb{R}^2 \) where \( M \) and \( N \) have continuous first partial derivatives. Then \( \mathbf{F} \) is conservative on \( D \) if and only if \( \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \) on \( D \). If \( \mathbf{F} = \langle M, N, P \rangle \) be a vector field on an open simply connected region \( D \) in \( \mathbb{R}^3 \) where \( M, N, \) and \( P \) have continuous first partial derivatives. Then \( \mathbf{F} \) is conservative on \( D \) if and only if \( \nabla \times \mathbf{F} = 0 \) on \( D \).

2. The integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) is independent of path on a region \( D \) if and only if \( \int_C \mathbf{F} \cdot d\mathbf{r} = 0 \) for every closed curve \( C \) in \( D \).

3. **Fundamental Theorem for Line Integrals** Let \( C \) be a piecewise smooth curve from a point \( P \) to a point \( Q \).

   If \( \mathbf{F} \) is a conservative vector field defined on \( C \) and \( f \) is a potential for \( \mathbf{F} \) then \( \int_C \mathbf{F} \cdot d\mathbf{r} = f \bigg|_Q^P = f(Q) - f(P) \).

4. **Green’s Theorem** Let \( C \) be a positively oriented, piecewise smooth, simple closed curve in \( \mathbb{R}^2 \) and let \( R \) be the region bounded by \( C \). If \( M(x, y) \) and \( N(x, y) \) have continuous first partial derivatives on an open region that contains \( R \) then \( \int_C M \, dx + N \, dy = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dA \).

5. If a surface \( S \) is given by \( \mathbf{r}(u, v) = \langle M(u, v), N(u, v), P(u, v) \rangle \) then \( \mathbf{r}_u \times \mathbf{r}_v \) is normal to \( S \).

6. If a surface \( S \) is given by \( \mathbf{r}(u, v) = \langle M(u, v), N(u, v), P(u, v) \rangle \) for \( \langle u, v \rangle \in D \) then the area of \( S \) is given by \( \iint_S d\mathbf{S} = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| \, dA \).

7. If a surface \( S \) is given by \( \mathbf{r}(u, v) = \langle M(u, v), N(u, v), P(u, v) \rangle \) for \( \langle u, v \rangle \in D \) and \( f(x, y, z) \) is a function defined on \( S \) then the surface integral of \( f \) over \( S \) is given by \( \iint_S f(x, y, z) \, d\mathbf{S} = \iint_D f(x, y, z) |\mathbf{r}_u \times \mathbf{r}_v| \, dA \).
8. If an oriented surface $S$ is given by $\mathbf{r}(u, v) = \langle M(u, v), N(u, v), P(u, v) \rangle$ for $(u, v) \in D$ and $\mathbf{F}$ is a continuous vector field defined on $S$ is $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iint_D (\mathbf{F} \cdot \mathbf{n}) |\mathbf{r}_u \times \mathbf{r}_v| \, dA$, where $\mathbf{n}$ is a unit normal vector for $S$.

The following is a list of things you should know and be able to do for the test. It is not necessarily complete.

1. Determine the image of a given curve under a transformation.
2. Evaluate the Jacobian of a transformation.
3. Use a given transformation and its Jacobian to evaluate a double or triple integral by a change of variables.
4. Determine which transformation should be used to simplify a double or triple integral by a change of variables.
5. Find the curl and divergence of a vector field.
6. Determine whether a vector field is conservative.
7. Find the potential of a conservative vector field.
8. Evaluate a line integral of the form $\int_C f \, ds$.
9. Evaluate a line integral of the form $\int_C \mathbf{F} \cdot d\mathbf{r}$.
10. Use a line integral to determine the work done by a force when an object moves from one point to another along a given curve.
11. Use the Fundamental Theorem of Line Integrals and the potential of conservative vector field to evaluate an integral of the form $\int_C \mathbf{F} \cdot d\mathbf{r}$.
12. Determine whether a line integral is independent of path.
13. Use the fact that a line integral is independent of path to evaluate the line integral from one point to another or along a given curve.
14. Use Green’s Theorem to evaluate a line integral.
15. Use Green’s Theorem to evaluate a double integral.
16. Find a parametrization for a surface.
17. Use the parametrization for a surface to find a normal vector to the surface.
18. Find the area of a parametric surface.
19. Evaluate a surface integral of the form $\iint_S f \, dS$.
20. Evaluate a surface integral of the form $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S (\mathbf{F} \cdot \mathbf{n}) \, dS$.