TOPICS AND EXPECTATIONS FOR MTH 219

The chapters and section numbers are those of *A First Course in Differential Equations, the Classic Fifth Edition*, by Zill

**Preliminary Theory:** (2.1) Know what a differential equation is, and what it means for a function to be a solution of a differential equation. Be familiar with the basic terminology of differential equations, such as order, ordinary differential equations, partial differential equations, linear and nonlinear differential equations. This material is presented in the first chapter of Zill. Know what an initial value problem is, and what it means for a function to be a solution of an initial value problem. Know the conditions under which an initial value problem will have a unique solution.

**First order differential equations:** (2.2–6) Know what it means for a first order differential equation to be separable, homogeneous, exact, linear, or Bernoulli. Recognize when a first order differential equations fits any of the above types. Be familiar with the techniques used to solve each of the above types of differential equations, and be able to apply the appropriate method to obtain a solution to a differential equation or an initial value problem.

**Higher order differential equations:** (4.1–7) Be familiar with the standard form of a linear differential equation of order higher than 2, and also with their initial value or boundary value problems. Know the conditions under which an initial value problem will have a unique solution. Know what it means for functions to be linearly dependent or independent. Know what the Wronskian of $n$ functions is and be able to find the Wronskian of two or three functions. Know what the connection is between the Wronskian of $n$ functions and whether or not they are linearly independent. Use the Wronskian to help decide whether a set of functions is linearly independent. Know what it means for a differential equation of order greater than 1 to be homogeneous. Know what a linear combination of functions is, and that every linear combination of solutions to a homogeneous linear differential equation is also a solution. Know that every homogeneous linear differential equation of order $n$ has $n$ linearly independent solutions, and that every solution of this equation is a linear combination of these $n$ solutions. Know what is meant by the complementary function or solution of a nonhomogeneous differential equation, and what is meant by a particular solution. Every solution of a nonhomogeneous linear differential equation can be written as the sum of its complementary function and a particular solution. Be able to construct a second solution for a homogeneous or nonhomogeneous linear differential equation from a given solution using the reduction of order method. Construct the auxiliary equation for a homogeneous linear differential equation. Solve such an equation. Know what type of solution a homogeneous differential equation has based on the type of solutions its auxiliary equation has. Construct the general solution of a homogeneous
linear differential equation from its auxiliary equation. Know what is meant be a linear 
operator, and be able to write a linear differential equation as a linear operator equation.
Be able to factor a linear operator. Know what is meant by the annihilator of a function
and be able to construct the annihilator of a function of the form $x^n$, $x^n e^{kx}$, $x^n \sin x$,
$x^n \cos x$, $x^n e^{kx} \sin bx$, or $x^n e^{kx} \cos bx$, where $k$ and $b$ are constants and $n$ is a nonnegative
integer. Use the superposition approach to the method of undetermined coefficients to
solve a nonhomogeneous linear differential equation. Use the Wronskian approach to the
variation of parameters method to solve a nonhomogeneous linear differential equation.

Applications: (5.1–4) Be familiar with the differential equations that represent simple
harmonic motions, including damped and undamped, forced and unforced. Be able to
solve these equations, and interpret the types of motion represented by the solutions.

Cauchy-Euler equation: (6.1) Know the general form of a Cauchy-Euler equation. Find
the auxiliary equation of a second or third order Cauchy-Euler equation, and know how
to find the solution of a homogeneous Cauchy-Euler equation from the solutions of the
auxiliary equation. Use variation of parameters to find the solution of a nonhomogeneous
Cauchy-Euler equation. Be able to transform a Cauchy-Euler equation to an equation with
constant coefficients, and use this equation to solve the original Cauchy-Euler equation.

Laplace transform: (7.1–6) Know the definition of the Laplace transform. Be able to
use the definition to find the Laplace transform of simple functions such as $f(t) = t$ or
$f(t) = e^{kt}$, and also of piecewise defined functions. Use a table of Laplace transforms to
find the inverse Laplace transform of basic functions. Use partial fraction expansion to
find the inverse Laplace transform of more complicated functions. Use the first translation
theorem to evaluate the Laplace transform of functions of the form $e^{at} f(t)$. Use the
second translation theorem to evaluate the Laplace transforms of functions that include
step functions. Know how derivatives are related to Laplace transforms, and use this
connection to find the Laplace transform of functions of the form $t^n f(t)$. Find the Laplace
transform of the derivative of a function. Know what the convolution of two functions
is, and how to find the Laplace transform of the convolution of two functions. Find the
Laplace transform of a periodic function. Use the Laplace transform to solve initial value
and boundary value problems, and integral equations. Be familiar with the Dirac delta
function and its Laplace transform.

Numerical methods: (9.2,4) Use Euler’s method and the Runge-Kutta method to obtain
approximations for the solutions of first order differential equations. Know how to find
absolute error, relative error, and percentage relative error.

Systems of differential equations: (8.3–6) Be familiar with the normal or canonical
form of a system of linear first-order equations. Be able to convert a differential equation
or a system of differential equations to a system of linear first-order differential equations.
Be familiar with the basic notation and terminology of matrices. Be able to perform ba-
sic arithmetic with matrices. Be able to take the transpose of a matrix. Know what it
means for a square matrix to be invertible or nonsingular. Be able to take the determi-
nant of a square matrix. Use the determinant of a matrix to decide whether the matrix
is invertible. Be able to find the inverse of an invertible $2 \times 2$ or $3 \times 3$ matrix. Take derivatives and integrals of matrices of functions. Convert a system of equations into a matrix. Know what the elementary row operations are and use them to solve a system of linear equations. Know what an eigenvalue and corresponding eigenvector are. Find the eigenvalues and corresponding eigenvectors of a $2 \times 2$ matrix. Convert a system of linear differential equations into a matrix. Know what a solution vector is. Know that every linear combination of solution vectors of a system of linear differential equations is again a solution. Know what it means for solution vectors to be linearly independent or dependent. Know the connection between linear independence and the Wronskian. Use the Wronskian to determine whether given solution vectors are linearly independent. Know that every homogeneous system of $n$ linear differential equations has a set of $n$ linearly independent solution vectors, and that every solution vector is a linear combination of these. Know what complementary and particular solutions are, and how to combine them to obtain a general solution. Use solution vectors to construct the fundamental matrix of a system of linear differential equations. Use eigenvalues and eigenvectors to solve a $2 \times 2$ system of linear differential equations. Be able to do this for distinct real, repeated real, and complex eigenvalues.