STUDY GUIDE FOR TEST 2 OF MTH 310

Test 2 is scheduled for Wednesday, November 2. It will cover sections I, II, and III.1–3 of Chapter 2 from your book. This material is on pages 77–128.

Be able to define the following terms.

1. subspace of a vector space
2. span of a set ([A])
3. linearly independent set
4. linearly dependent set
5. basis
6. dimension of a vector space
7. row space of a matrix
8. column space of a matrix
9. rank of a matrix
10. transpose of a matrix (AT)

Be familiar with the following theorems.

1. I.2,9,15
2. II.1.4,12,14,16
3. III.1.12
4. III.2.4,10,11,12,13
5. III.3.3,4,10,11,13,15

The following is a list of things you should know and be able to do for the test. It is not necessarily complete.

1. Know the definition of a vector space. When a set with addition and scalar multiplication is not a vector space identify the properties of a vector space which fail to hold. Show that selected properties of a vector space do hold for a given set with addition and scalar multiplication.
2. Prove simple properties of the operations of a vector space, such as those in Lemma I.1.14 of Chapter 2.
3. Determine whether or not a given set is a subspace of a vector space.
4. Find the span of a subset of a vector space.
5. Determine whether a given vector is in the span of a set.
6. Determine whether a set of vectors is linearly dependent or independent.
7. Determine whether a linearly independent set will remain linearly independent when a given vector is added to the set.
8. Given a subset A of a vector space find B ⊆ A such that B is linearly independent and |B| = |A|.
9. Determine whether a set is a basis for a vector space.
10. Find the representation of a vector with respect to a basis.
11. Expand a given linearly independent set into a basis.
12. Given a spanning subset A of a vector space V, find a subset of A that is a basis for V.
13. Find a basis for the row space of a matrix.
14. Find a basis for the column space of a matrix.
15. Determine whether a vector is in the row space of a matrix.
16. Determine whether a vector is in the column space of a matrix.
17. Determine the rank of a matrix.

18. Know the connection between the rank of a square matrix $A$, its singularity, and the number of solutions of homogeneous system having $A$ as its coefficient matrix.

19. Prove simple properties of span, linear independence, bases, and dimension.