LSN 4 – Key Terms

- **Variable**: a symbol used to represent a logic quantity
- **Compliment**: the inverse of a variable
- **Literal**: a variable or compliment

\[(A + B + C)D\]
LSN 4 – Boolean Operations

• Boolean addition
  – Sum term:
    • Sum of all literals
    • $A + B, \overline{A} + \overline{B}, A + B + \overline{C} + D$
    • Equal to 1 when one or more literals in the term are 1
    • Equal to 0 only if all literals are 0

• Boolean multiplication
  – Product term
    • Product of all literals
    • $AB, AB, ABCD$
    • Equal to 1 only if each literal is 1
    • Equal to 0 if one or more literals is 0
LSN 4 – Laws of Boolean Algebra

- Communicative laws
  - For addition
    \[ A + B = B + A \]
  
  - For multiplication
    \[ AB = BA \]
LSN 4 – Laws of Boolean Algebra

- **Associative laws**
  - For addition
    \[ A + (B + C) = (A + B) + C \]
    ![Diagram](A plus B plus C equals A plus B plus C)

  - For multiplication
    \[ A(BC) = (AB)C \]
    ![Diagram](A times B times C equals A times B times C)
LSN 4 – Laws of Boolean Algebra

- Distributive laws

\[ A(B + C) = AB + AC \]

or

\[ AB + AC + AD = A(B + C + D) \]
LSN 4  – Boolean Algebra Rules

- \( A + 0 = A \)

- \( A + 1 = 1 \)

- \( A \cdot 0 = 0 \)
LSN 4  – Boolean Algebra Rules

• \( A \cdot 1 = A \)

\[
\begin{align*}
A = 0 & \quad \text{X} = 0 & \quad A = 1 & \quad \text{X} = 1 \\
1 & \quad & 1 & \\
\end{align*}
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• \( A + A = A \)

\[
\begin{align*}
A = 0 & \quad \text{X} = 0 & \quad A = 1 & \quad \text{X} = 1 \\
A = 0 & \quad & A = 1 & \\
\end{align*}
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• \( A + \overline{A} = 1 \)

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\begin{align*}
A = 0 & \quad \text{X} = 1 & \quad A = 1 & \quad \text{X} = 1 \\
\overline{A} = 1 & \quad & \overline{A} = 0 & \\
\end{align*}
\]
LSN 4 – Boolean Algebra Rules

- \( A \cdot A = A \)

- \( A \cdot \overline{A} = 0 \)

- \( \overline{A} = A \)
LSN 4  – Boolean Algebra Rules

• $A + AB = A$

• $A + \overline{A}B = A + B$

• $(A + B)(A + C) = A + BC$
LSN 4 – Demorgan’s Theorems

• The compliment of a product of literals is equal to the sum of the compliments of the literals

\[ \overline{XY} = \overline{X} + \overline{Y} \]
LSN 4 – Demorgan’s Theorems

• The compliment of a sum of literals is equal to the product of the compliments of the literals

\[ \overline{X + Y} = \overline{X} \overline{Y} \]

– Can be applied to groupings of literals, not just individual literals

\[ (A + B + C)D = (A + B + C) + \overline{D} = \overline{ABC} + \overline{D} \]
LSN 4 – Demorgan’s Theorems

• Example:
  – Exclusive-OR

\[ x = A\overline{B} + \overline{A}B \]

  – Find logical expression for Exclusive-NOR
LSN 4 – Demorgan’s Theorems

• Example:
  – Apply DeMorgan’s theorems to
    \[ X = ABC + DEF \]
    
    \[ X = \overline{AB} + \overline{CD} + EF \]
LSN 4 – Boolean Analysis of Circuits

• Logic analysis
  – Start at left-most inputs and work towards the final output
    • Create an expression for each gate
    • Example:

![Logic Circuit Diagram]

A
B
C
D
E
X
Use Boolean algebra techniques to simplify the expression and the circuit

Example:

\[ X = AB + AC + ABC \]
LSN 4 – Standard Form Expressions

- **Sum of products (SOP)**
  - \( \overline{A}B + \overline{B}C, AC + AB + CD, A + B\overline{C} \)

- **Product of Sums (POS)**
  - \((A + \overline{B})(B + C), (\overline{A} + B)C\)  

- **Key terms:**
  - Domain: all variables/compliments present in a Boolean expression
    - \( X = AB + C \)
    - domain = A, B, and C
LSN 4 – Standard Form Expressions

• Standard SOP
  – The domain is completely represented in each product term
    \[ X = \bar{A}BC + A\bar{B}C + A\bar{B}\bar{C} \]
  – Create the standard SOP form
    • Recall \( A + \bar{A} = 1 \)
    • Multiply each product by the appropriate \( (A + A) \) term to achieve standard SOP form
    • Example:
      \[ X = AB + C \]
LSN 4 – Standard Form Expressions

• Standard POS
  – The domain is completely represented in each sum term
    \[ X = (A + B + \overline{C})(A + \overline{B} + C) \]
  – Create the standard POS form
    • Recall \( \overline{A}A = 0 \)
    • Add appropriate \( \overline{A}A \) term to each non-standard product term
    • Example:
      \[ X = (A + B)C \]
LSN 4 – Expressions & Truth Tables

- SOP expression is equal to 1 only if at least one of the product terms equal 1

\[ X = \bar{A}BC + AB\bar{C} + ABC \]

Binary value

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<th>A</th>
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LSN 4 – Expressions & Truth Tables

- POS expression is equal to 0 only if at least one of the sum terms equal 0

\[ X = (A + B + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C}) \]

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Binary value
LSN 4 – Expressions & Truth Tables

- Determine standard SOP and POS forms from truth table
  - Reverse process
    - Where a 1 exists for X, the resulting product terms form the standard SOP expression
    - Where a 0 exists for X, the resulting sum terms form the standard POS expression

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SOP    POS
LSN 4 – Karnaugh Map

• Representation of all possible inputs and resulting outputs for a logic expression
• Used to simplify a Boolean expression to its minimal form
LSN 4 – Karnaugh Map

- Cells are arranged so only one variable changes between adjacent cells
  - This is referred to as “Cell Adjacency”
LSN 4 – Karnaugh Map SOP Minimization

• Mapping a standard form SOP expression
  – Place a 1 in the Map for each product term
  – Example:
    \[ X = \overline{A}BC + ABC + A\overline{B}\overline{C} \]
LSN 4 – Karnaugh Map SOP Minimization

• Mapping a nonstandard SOP expression
  – Recall multiplying product terms by necessary \((A + \bar{A})\) combinations to achieve standard form
  – Numerical expansion method
    • Determine the binary form for each product term
    • Where a binary value is missing, insert a 0 and duplicate the original binary form with a 1 inserted for the missing binary term
  • Example:

\[
X = \bar{A}B + ABC
\]

\[
\begin{array}{cccc}
1 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 \\
\end{array}
\]

- Missing term
- Insert 0
- Duplicate binary form
- Binary expanded numbers

- Insert 1

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LSN 4 – Karnaugh Map SOP Minimization

• Minimization process
  – Group all 1s in K-Map
    • Must be in groupings of a power-of-2 size (1, 2, 4, 8, 16)
    • Maximize group size
    • Group only adjacent cells
    • Each 1 in K-Map must belong to at least one group
  – Determine product terms for groupings
    • Composed of variables that occur in only one form within the grouping
  – Sum the resulting product terms
LSN 4 – Karnaugh Map SOP Minimization

- Example:
LSN 4 – Karnaugh Map POS Minimization

• Mapping a standard form expression
  – Place a 0 in the Map for each sum term
  – Example:
    \[ X = (A + B + C)(\bar{A} + \bar{B} + C) \quad (A + \bar{B} + \bar{C}) \]
LSN 4 – Karnaugh Map POS Minimization

• Minimization process
  – Group all 0s in K-Map
    • Follow same rules used to group all the 1s for simplification of SOP expressions
  – Determine sum terms for groupings
  – Multiply the resulting sum terms
LSN 4 – Karnaugh Map

• Example:

$$(B + C + \overline{D}) (A + \overline{B} + C + D) (A + \overline{B} + \overline{C} + \overline{D}) (\overline{A} + \overline{B} + C + D) (\overline{A} + \overline{B} + \overline{C} + \overline{D})$$
LSN 4 – Homework

• Reading
  – Chapter 4.1 – 4.4
  – Chapter 4.6 – 4.10

• Assignment – HW5
  – Chapter 4, problems 5(d, g), 6(c, e), 10, 15(a, d), 21(a, c, e)
    • Show all work for credit

• Assignment – HW6
  – Chapter 4, problems 23, 25, 33, 36, 44, 46
    • Show all work for credit