1. (10 points) Sketch the curve \( C \) determined by \( \mathbf{r}(t) = (1-t^3)\hat{i}+t\hat{j}, \ t \geq 0 \), and determine the orientation of the curve.

2. (10 points) Find the length of the parameterized curve \( x = 2\cos(t), \ y = 2\sin(t), \ z = 3t, \ 0 \leq t \leq 2\pi \).

3. (10 points) Find the domain of \( \mathbf{r}(t) = \ln(t-1)\hat{i} - e^{t^2}\hat{j} + \sqrt{5-t}\hat{k} \).
4. (16 points) Evaluate:

(a) \( \lim_{t \to 0} \left( \frac{\sin(t)}{t} \hat{i} + \cos(t) \hat{j} - e^t \hat{k} \right) \)

(b) \( \int \left( te^{t^2} \hat{i} + \sqrt{t} \hat{j} + \frac{1}{t^2 + 1} \hat{k} \right) dt \)

5. (15 points) If \( \vec{r}(t) = e^t \left( \cos(t) \hat{i} + \sin(t) \hat{j} + t \hat{k} \right) \) is the position vector of a moving point \( P \), find the velocity, acceleration, and the speed at \( t = \frac{\pi}{2} \).
6. (10 points) Find the curvature and the radius of curvature of the curve \( y = 2 - x^3 \) at \( P(1, 1) \).

7. (19 points) Let \( \vec{r}(t) = 2 \sin(t)\hat{i} + 3 \cos(t)\hat{j} \). Find:
   
   (a) unit tangent vector to the curve.
   
   (b) principal normal vector to the curve.
   
   (c) the tangential component of the acceleration at \( P(1, \frac{3\sqrt{3}}{2}) \).
(d) the normal component of the acceleration at $P(1, \frac{3\sqrt{3}}{2})$.

(e) curvature of curve at $P(1, \frac{3\sqrt{3}}{2})$.

8. (10 points) A baseball player throws a ball a distance of 250 feet. If the ball is released at angle of $45^\circ$ with the horizontal, find its initial speed.

9. (5 Bonus points) If a point moves at a constant speed, prove that the velocity and the acceleration vectors are orthogonal.