Chapter 13
Reliability Testing

13.1 Product Testing
13.2 Reliability Life Testing
13.3 Test Time Calculations
13.4 Burn-in & screen testing
13.6 Accelerated life testing
Test Design

• Test objective
  • e.g. reliability demonstration, reliability improvement, screening
• Type of test
  • e.g. sequential, accelerated
• Operating & environmental conditions
• Number of units to be tested (sample size)
• Duration of test
  • failure terminated vs time terminated
• Definition of a failure
Cumulative Time on Test - CFR

n = nbr on test
r = nbr failures
k = nbr multiply censors
t_i = failure time
t_i^+ = censor time
t_* = test time (Type I)
t_r = test time (Type II)

MTTF = T / r

Complete: \[ \sum_{i=1}^{n} t_i; \quad r = n \]

Type I: \[ \sum_{i=1}^{n} t_i + (n-r)t_* \]

Type II: \[ \sum_{i=1}^{n} t_i + (n-r)t_r \]

Type I multiply: \[ \sum_{i=1}^{n} t_i^+ + (n-r-k)t_* \]

Type II multiply: \[ \sum_{i=1}^{n} t_i^+ + (n-r-k)t_r \]

Type I replacement: \[ nt_* \]

Type II replacement: \[ nt_r \]
Example 13.1

During a testing cycle, 20 units were tested for 50 hours with the following failure times and censor times observed:

10.8, 12.6+, 15.7, 28.1, 30.5, 36.0+, 42.1, 48.2

For Type I testing with $t^* = 50$ hours as the test termination time,

$$T = 10.8 + 12.6 + 15.7 + 28.1 + 30.5 + 36.0 + 42.1 + 48.2 + (20 - 6 - 2) \times 50 = 824 \text{ hours}$$

Then $\text{MTTF} = 824/6 = 137.3 \text{ hours.}$
Example 13.2

Ten units were placed on test with a failed unit immediately replaced. The test was terminated after the 8th failure which occurred at 20 hours.

This is type II testing with replacement. Therefore

\[ T = (10)(20) = 200 \text{ hours} \]

\[ \text{MTTF} = \frac{200}{8} = 25 \text{ hours} \]
with replacement:

\[ \lambda = \text{failure rate of single unit} \]

\[ n \lambda = \text{system failure rate with } n \text{ units operating} \]

\[ 1/(n \lambda) = \text{MTTF}/n = \text{system MTTF} \]

\[ E(\text{test time}) = r \times \text{MTTF} / n \]
without replacement: generate $r$ failures:

With $n$ units on test: system MTTF = $\frac{MTTF}{n}$
With $n-1$ units on test: system MTTF = $\frac{MTTF}{(n-1)}$
With $n-2$ units on test: system MTTF = $\frac{MTTF}{(n-2)}$

With $n-r+1$ units on test: system MTTF = $\frac{MTTF}{(n-r+1)}$

$$E(test\ time) = MTTF \times TTF_{r,n} = MTTF \left[ \frac{1}{n} + \frac{1}{n-1} + \ldots + \frac{1}{n-r+1} \right]$$
Example 13.3

To support the current cycle in a reliability growth testing program, a total of 8 failures need to be generated. The current estimate of the MTTF is 55 hours. The test department is scheduled to complete testing within 72 hours. How many units should be placed on test?

This is Type II testing. Since the length of the test is MTTF x TTF then the $TTF_{8,n} = 72/55 = 1.31$. From the table,

- $TTF_{8,10} = 1.429$
- $TTF_{8,11} = 1.187$
Example 13.4

For the problem in Example 13.3, the test department is told they must complete the testing within 48 hours. How many failures would they expect to generate?

\[ E(r) = 11 \left( 1 - e^{-\frac{48}{55}} \right) = 6.4 \text{ units} \]
Burn-in Testing

Goal is to increase the mean residual life

Eliminate “early” customer failures by generating test hours in the factory

There must be a dominate DFR failure mode

Primary question is how long to test?

Specification model

cost model
Given a reliability goal of $R_0$ where $R(t_0) < R_0$ and $R(t)$ has a DFR, then a burn-in period, $T$, is desired such that $R(t_0 | T) = R_0$.

For the Weibull distribution

$$R(t_0 | T) = \frac{e^{-\left(\frac{t_0+T}{\theta}\right)^\beta}}{e^{-\left(\frac{T}{\theta}\right)^\beta}} = R_0$$

$$e^{-\left(\frac{t_0+T}{\theta}\right)^\beta} - R_0 e^{-\left(\frac{T}{\theta}\right)^\beta} = 0$$
Burn-in Testing

\[ \text{burn-in failures} \]

\[ R(t_0) < R_0 \]

\[ t_0 \]

\[ T \]

\[ t \]
Example 13.5

Reliability testing has shown that a ground power unit used to supply DC power to aircraft has a Weibull distribution with beta = .5 and theta = 45,000 operating hours. Determine a burn-in period necessary to obtain a required reliability specification of R(1000) = .90.

Observe that R(1000) = .86 and beta < 1.

\[- \left(1 + \frac{T}{45000}\right)^{.5} e^{\frac{T}{45000}} - 0.90 e^{\frac{T}{45000}} = 0\]

\[-(1000 + T)^{.5} e^{\frac{T}{45000}} - 0.90 e^{\frac{T}{45000}} = 0\]

\[T^* = 126 \text{ hr. and } R(1000|126) = 0.90\]
Burn-in Testing - Cost Model

\( C_b = \) cost per unit time for burn-in testing  
\( C_f = \) cost per failure during burn-in  
\( C_o = \) cost per failure when operational  
\( T = \) length of burn-in testing  
\( t = \) operational life of the units  

\[
E[C(T)] = C_b T + C_f [ 1 - R(T)] + C_o [ R(T) - R(t+T) ] 
\]

for the Weibull:

\[
E[C(T)] = C_b T + C_f \left[ 1 - e^{-\left(\frac{T}{\theta}\right)^\beta} \right] + C_o \left[ e^{-\left(\frac{T}{\theta}\right)^\beta} - e^{-\left(\frac{T+t}{\theta}\right)^\beta} \right] 
\]
Example 13.6

The replacement cost on a new product if it fails during its operational life of 10 years (3650 days) is $6200. It will cost the company $70 a day per unit tested to operate a burn-in program and any failures during burn-in will cost $500. Reliability testing has established the life distribution of the product to be Weibull with beta = .35, and theta = 3500 days. What is the minimum cost time period for the burn-in?

\[
E[C(T)] = 70T + 500\left[1 - e^{-\left(\frac{T}{3500}\right)^{.35}}\right] + 6200\left[e^{-\left(\frac{T}{3500}\right)^{.35}} - e^{-\left(\frac{T+3650}{3500}\right)^{.35}}\right]
\]

\[T^* = 1.9 \text{ days with } E[C(T)] = $3690\]
Example 13.6

\[ E[C(T)] = \$3952 \]

\[ T = 0, \ E[C(T)] = \$3952 \]

\[ E[C(T)] = \$3704 \]

\[ E[C(T)] = \$3690 \]
Accelerated Life Testing

Problem: test time < expected lifetimes

Solution:

- Increase the number of units on test (compressed time)
- Accelerate the number of cycles per unit of time
- Increase the stresses that generate failures (accelerated stress testing)
Increase Number of Units on Test

For CFR:

\[ f_{r,n} = \frac{TTF_{n,r}}{TTF_{r,r}} \]

without replacement

\[ f_{r,n} = r/n \]

with replacement

then

\( f_{r,n} \) is the factor reduction in expected test time

\( 100(1-f_{r,n}) \) is the percent savings in expected test time
Increase Number of Units on Test

for Weibull:

$$f_{r,n} = \left( \frac{TTF_{n,r}}{TTF_{r,r}} \right)^{1/\beta}$$
without replacement

$$f_{r,n} = \left( \frac{r}{n} \right)^{1/\beta}$$
with replacement

then

$f_{r,n}$ is the factor reduction in expected test time
$100(1-f_{r,n})$ is the percent savings in expected test time
Example 13.10

Given \( n = 15 \) and \( r = 8 \):

**without replacement**  \quad **with replacement**

CFR model:  
\[ f_{8,15} = \frac{\text{TTF}_{8,15}}{\text{TTF}_{8,8}} \]
\[ f_{8,15} = \frac{8}{15} = 0.533 \]
\[ = \frac{0.725}{2.718} = 0.2667 \]

Weibull (Beta = 2)  
\[ f_{8,15} = \left(\frac{0.725}{2.718}\right)^{1/2} \]
\[ = 0.516 \]
\[ f_{8,15} = (\frac{8}{15})^{1/2} = 0.730 \]

Replacing vs not replacing failed units:  
\[ \frac{r}{n\text{TTF}_{r,n}} = \frac{8}{15(0.725)} = 0.7356 \]
Accelerated Cycling

for products that do not operate continuously or nearly continuously

\[ x_n = \text{the nbr of cycles per unit of time under normal cycling,} \]
\[ x_s = \text{the nbr of cycles per unit of time under accelerated cycling,} \]
\[ t_n = \text{time to failure under } x_n \text{ cycles per unit of time} \]
\[ t_s = \text{time to failure under } x_s \text{ cycles per unit of time} \]

\[ x_n t_n = x_s t_s \]

e.g. 10 cycles/hr x 100 hrs = 50 cycles/hr x 20 hrs = 1000 cycles

\[ R_n(t_n) = R_s(t_s) = R_s \left( \frac{x_n}{x_s} t_n \right) \]
An Observation

Let $T_s = \text{a random variable, the time to failure under accelerated cycling}$

$T_n = \text{a random variable, the time to failure under normal cycling}$

$$T_s = \frac{x_n}{x_s} T_n \quad \text{and} \quad E[T_s] = E \left[ \frac{x_n}{x_s} T_n \right] = \frac{x_n}{x_s} E[T_n] = \frac{x_n}{x_s} \mu_n$$

$$Var[T_s] = Var \left[ \frac{x_n}{x_s} T_n \right] = \left( \frac{x_n}{x_s} \right)^2 Var[T_n] = \left( \frac{x_n}{x_s} \right)^2 \sigma_n^2$$
for Weibull:

\[ R_n(t_n) = e^{-\left(\frac{t_n}{\theta_n}\right)^{\beta_n}} = e^{-\left(\frac{t_s}{\theta_s}\right)^{\beta_s}} = e^{-\left(\frac{x_n t_n}{x_s \theta_s}\right)^{\beta_s}} \]

\[ B_s = B_n = B \text{ and } \theta_n = \frac{x_s}{x_n} \theta_s \]
Example 13.11

An automotive part was tested at an accelerated cycling level of 100 cycles per hour. The resulting failure data was found to have a Weibull distribution with \( \beta = 2.5 \) and \( \theta = 1000 \) hours.

If the normal cycle time is 5 per hour, then

\[
\theta_n = \frac{100}{5} \times 1000 = 20,000 \text{ hr} \quad \text{and} \quad R_n(t) = e^{-\left(\frac{t}{20000}\right)^{2.5}}
\]
\[ R_n(t_n) = 1 - \Phi \left( \frac{t_n - \mu_n}{\sigma_n} \right) = 1 - \Phi \left( \frac{t_s - \mu_s}{\sigma_s} \right) \]

\[ = 1 - \Phi \left( \frac{x_n t_n - \mu_s}{x_s \sigma_s} \right) = 1 - \Phi \left( \frac{t_n - \frac{x_s}{x_n} \mu_s}{\frac{x_s}{x_n} \sigma_s} \right) \]

therefore: \( \mu_n = \frac{x_s}{x_n} \mu_s \) and \( \sigma_n = \frac{x_s}{x_n} \sigma_s \)
Accelerated Cycling and the LogNormal Distribution

\[
R_n(t_n) = 1 - \Phi \left( \frac{1}{s_n} \ln \frac{t_n}{t_{med,n}} \right) = 1 - \Phi \left( \frac{1}{s_s} \ln \frac{t_s}{t_{med,s}} \right) \\
= 1 - \Phi \left( \frac{1}{s_s} \ln \frac{x_n t_n}{x_{med,n}} \right) = 1 - \Phi \left( \frac{1}{s_s} \ln \frac{t_n}{x_{med,s}} \right)
\]

therefore: \( t_{med,n} = \frac{x_s}{x_n} t_{med,s} \) and \( s_n = s_s \)
Basic assumption: same failure mechanisms will be present at the higher stress levels and will act in the same manner as at normal stress levels.

\[ t_n = \text{time to failure under normal stress} \]
\[ t_s = \text{time to failure at high stress level} \]

then \[ t_n = AF \times t_s \] where AF is an acceleration factor

Therefore

\[ \Pr\{T_n < t_n \} = F_n(t_n) = \Pr\{T_s < t_s \} = F_s\left(\frac{t_n}{AF}\right) \]
## Failure Mechanisms and Stresses

<table>
<thead>
<tr>
<th>Failure Mechanism</th>
<th>Accelerated Stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal crack propagation</td>
<td>Temperature, dissipated power</td>
</tr>
<tr>
<td>Corrosion</td>
<td>Temperature and humidity</td>
</tr>
<tr>
<td>Cyclical (mechanical) fatigue</td>
<td>Cyclical vibration (stress amplitude and load frequency)</td>
</tr>
<tr>
<td>Thermal fatigue</td>
<td>Temperature cycling</td>
</tr>
<tr>
<td>Electro migration</td>
<td>Current density &amp; temperature</td>
</tr>
<tr>
<td>Stress-strength</td>
<td>Critical load and load frequency</td>
</tr>
<tr>
<td>Adhesive or abrasive wear</td>
<td>Loading, surface area, duration</td>
</tr>
</tbody>
</table>

**Test plans:**

1. What type of stresses?
2. What stress levels?
3. What number to test at each stress level?
Example 13.12

For the CFR model, a component is tested at 120°C and found to have an MTTF = 500 hours. Normal use is at 25°C. Assuming AF = 15, determine the components MTTF and reliability function at normal stress levels.

\[
F_n(t) = F_s\left(\frac{t}{AF}\right) = 1 - e^{-\lambda_s\left(\frac{t}{AF}\right)} = 1 - e^{-\frac{t}{500 \times 15}}
\]

\[
R(t) = e^{-\frac{t}{7500}} \quad \text{and MTTF} = 7500 \text{ hr.}
\]
Weibull Case

\[ F_s(t) = 1 - e^{-\left(\frac{t}{\theta_s}\right)^{\beta_s}} \]

\[ F_n(t) = 1 - e^{-\left(\frac{t}{AF\theta_s}\right)^{\beta_s}} \]

\[ \theta_n = AF\theta_s \quad \text{and} \quad \beta_n = \beta_s. \]

estimate \( AF \) by:

\[ AF = \frac{\theta_n}{\theta_s} \]
An Approach

Location parameter \((\theta, 1/\lambda, t_{med})\)

Least-squares fit
Example 13.15

Accelerated life testing for fracture stress failures at high temperature was conducted on 40 units at four different accelerated stress levels with the following results in hours:

<table>
<thead>
<tr>
<th>stress level*</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample</td>
<td>time to fail</td>
<td>time to fail</td>
<td>time to fail</td>
<td>time to fail</td>
</tr>
<tr>
<td>1.0</td>
<td>4355</td>
<td>1677</td>
<td>784</td>
<td>1067</td>
</tr>
<tr>
<td>2.0</td>
<td>4951</td>
<td>4707</td>
<td>1813</td>
<td>681</td>
</tr>
<tr>
<td>3.0</td>
<td>7503</td>
<td>2407</td>
<td>2240</td>
<td>1339</td>
</tr>
<tr>
<td>4.0</td>
<td>1475</td>
<td>1367</td>
<td>2731</td>
<td>1151</td>
</tr>
<tr>
<td>5.0</td>
<td>4703</td>
<td>5709</td>
<td>1533</td>
<td>1436</td>
</tr>
<tr>
<td>6.0</td>
<td>5002</td>
<td>2976</td>
<td>1639</td>
<td>507</td>
</tr>
<tr>
<td>7.0</td>
<td>4091</td>
<td>2343</td>
<td>1979</td>
<td>1174</td>
</tr>
<tr>
<td>8.0</td>
<td>2514</td>
<td>2879</td>
<td>1612</td>
<td>655</td>
</tr>
<tr>
<td>9.0</td>
<td>4775</td>
<td>3551</td>
<td>1233</td>
<td>1156</td>
</tr>
<tr>
<td>10.0</td>
<td>1898</td>
<td>2804</td>
<td>1705</td>
<td>773</td>
</tr>
</tbody>
</table>

*stress levels are measured in psi
Example 13.15

<table>
<thead>
<tr>
<th>stress level</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>2.7</td>
<td>2.5</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>4647</td>
<td>3445</td>
<td>1938</td>
<td>1117</td>
</tr>
</tbody>
</table>

![Graph showing linear regression](ImageOfGraph.png)

The linear regression equation is:

$y = -120.97x + 13069$

$R^2 = 0.9885$
Example 13.15

At the normal stress level of 2 psi, $\theta = 12,827$ hr. Using the average of the four $\beta$ values, the reliability model at the normal stress level (2 psi) becomes:

$$\theta = y = -120.97(2) + 13069 = 12,827$$

$$\beta = \frac{2.7 + 2.5 + 3.2 + 3.1}{4} = 2.875$$

$$R(t) = e^{-\left(\frac{t}{12,827}\right)^{2.875}}$$
Nonlinear stress effects

Assume \( t_n = k \tau_s^m \), \( m \neq 1 \).

\[
F_n(t_n) = 1 - \exp \left[ -\left( \frac{t_n}{\theta_n} \right)^{\beta_n} \right] = 1 - \exp \left[ -\left( \frac{t_n^{1/m}}{k \cdot \theta_s} \right)^{\beta_s} \right] = 1 - \exp \left[ -\left( \frac{t_n}{(k \cdot \theta_s)^m} \right)^{\beta_s/m} \right]
\]

both the scale and shape parameters change
Arrhenius Model
used when increased temperature is the applied stress

\[ r = A e^{\frac{-B}{T}} \]

where \( r \) is the reaction or process rate, \( A \) and \( B \) are constants, and \( T \) is temperature measured in degrees Kelvin.

\[ AF = \frac{A e^{\frac{-B}{T_2}}}{A e^{\frac{-B}{T_1}}} = e^{B \left( \frac{1}{T_1} - \frac{1}{T_2} \right)} \]

\[ B = \frac{\ln AF}{\left( \frac{1}{T_1} - \frac{1}{T_2} \right)} \text{ where } AF = \frac{\theta_1}{\theta_2} \]
Example 13.14

An electronic component has a normal operating temperature of 294° K (about 21° C). Under stress testing at 430° K a Weibull distribution was obtained with theta = 254 hours, and at 450° K., a Weibull distribution was obtained with theta = 183 hours. The shape parameter did not change with beta = 1.72.

\[
B = \ln\left(\frac{254}{183}\right) = 3172
\]

\[
AF = e^{3172\left(\frac{1}{294} - \frac{1}{450}\right)} = 42.1
\]

Weibull with a shape parameter of 1.72 and \( \theta = 42.1 \times 183 = 7704.3 \) hours.
Eyring Model

\[ r = A T^a e^{\frac{B}{T}} e^{CS} \]

A, \( \alpha \), B, and C are constants to be estimated

\[ AF = \left( \frac{T_2}{T_1} \right)^a e^{B \left( \frac{1}{T_1} - \frac{1}{T_2} \right)} e^{C(S_2 - S_1)} \]
Degradation Models

\[ y = a - b \, t, \quad \text{where } y \text{ is the performance measure, } a \text{ and } b \text{ are constants to be determined experimentally, and } t \text{ is the amount of time the product is exposed at a constant stress level} \]

\[
t_f = \frac{a - y_f}{b}
\]

where \( y_f \) is the performance level at which a failure occurs.
Example 13.15

\[ CPR = \frac{kw(t)}{\rho At} \]

- \( t \) = exposure time in hours,
- \( w(t) \) = weight loss due to corrosion after \( t \) hours exposure in mg
- \( \rho \) = density of the material in grams per cubic centimeter
- \( A \) = exposed surface area in square centimeters
- \( k = 87.6 \), a constant which converts CPR to millimeters per year

If \( l_f \) is the allowable loss in millimeters after which the material is no longer structurally sound, then the time to failure is projected to be

\[ t_f = \frac{l_f}{CPR} \]
Example 13.16

\[ p = e^{-rt} \]

where \( p \) = potency of the drug
\( r \) = rate of chemical reaction
\( t \) = drug exposure time

then \( t = -\ln p / r \)

\[ r = Ae^{\frac{-B}{T}} \text{ then } t = \frac{-\ln p}{Ae^{-\frac{B}{T}}} \]
Minor’s rule:

\[
\sum_{i=1}^{n} \frac{t_i}{L_i} = 1
\]

\(t_i\) = the amount of time at stress level \(i\)

\(L_i\) = the expected lifetime at stress level \(i\)

\[
\frac{t_1}{L_1} + \frac{t_2}{L_2} = 1 \text{ or } t_2 = L_2 - \frac{L_2}{L_1} t_1
\]
Cumulative Damage Models

1. test to failure

2. test to \( t_1 \) then to failure \( t_2 \)

3. solve

\[
L_1 = \frac{t_1}{1 - \frac{t_2}{L_2}}
\]
I have found this discussion to be quite stress full

I feel I have degraded in an accelerated way!