A z score is a way of standardizing the scale of two distributions. When the scales have been standardized, it is easier to compare scores on one distribution to scores on the other distribution.

An Example

You scored 80 on exam 1 and 75 on exam 2. On which exam did you do better?

The answer may or may not be that you did better on exam 2. In order to decide on which exam you did better, you must also know the mean and standard deviation of the exams.
An Example

® The mean and standard deviation of Exam 1 were 85 and 5, respectively
® The mean and standard deviation of Exam 2 were 70 and 5, respectively
® So, you scored below the mean on exam 1 and above the mean on exam 2
® On which exam did you do better?

z Scores

® A z score is defined as the deviate score (the observed score minus the mean) divided by the standard deviation
® It tells us how far a score is from the mean in units of the standard deviation

\[ Z = \frac{(X - \mu)}{\sigma} \]

An Example

® You have a z score of -1 on the first exam
  ® Your score was one standard deviation below the mean on exam 1
® You have a z score of 1 on the second exam
  ® Your score was one standard deviation above the mean on exam 2
® You did better on exam 2
Interpretation of the z Score

- z-scores consist of a sign and a magnitude
- If the sign is negative, the score is below the mean; if the sign is positive, the score is above the mean
- The magnitude tells you have many standard deviations the score is above or below the mean

Important Properties of z Scores

- The mean of a distribution of z scores is always 0
- The standard deviation of a distribution of z scores is always 1
- The sum of the squared z scores always equals N

Proofs

\[ \mu_z = 0 \]
\[ \sigma_z = 1 \]
\[ \sum z^2 = N \]
z Score for Samples

\[ Z = \frac{(X - \bar{X})}{s} \]

Raw Score from z Score

\[ X = \mu + z \cdot \sigma \]
\[ X = \bar{X} + z \cdot s \]