Two Factor ANOVA

Factorial Designs

- Often researchers want to study the effects of two or more independent variables at the same time.
- Does it matter where a list of words is studied, on the beach or under water?
- Does it matter where a list of words is recalled, on the beach or under water?

Factorial Designs

- *Factor* is another name for independent variable
- The preceding example has two factors: where you study and where you recall
- In a factorial design, all possible combinations of the factors are present
A Factorial Design

<table>
<thead>
<tr>
<th>Recall</th>
<th>Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>On Beach</td>
<td>Study on beach; recall on beach</td>
</tr>
<tr>
<td>Underwater</td>
<td>Study on beach; recall underwater</td>
</tr>
<tr>
<td>On Beach</td>
<td>Study underwater; recall on beach</td>
</tr>
<tr>
<td>Underwater</td>
<td>Study underwater; recall underwater</td>
</tr>
</tbody>
</table>

Information From A Factorial Design

- A two factor design is powerful because it answers three questions:
  - Is there an effect of the first IV?
    - Do you recall more words when you study them on the beach or underwater?
  - Is there an effect of the second IV?
    - Do you recall more words when you recall them on the beach or underwater?
  - Are the effects of the two IVs independent of each other?
    - When recalling on the beach, does it matter whether you studied underwater or not? When recalling underwater, does it matter whether you studied underwater or not?

Main Effects

- Each of the first two questions (Is there an effect of the first / second IV?) is asking whether there is a main effect of that IV.
- The mean differences among the levels of one factor are referred to as the main effect of that factor.
  - A main effect occurs when an IV influences the DV
    - If people recalled more words they studied on the beach than they studied underwater, then there would be a main effect of where the words are studied.
Main Effects

1. **H₀**: \( \mu_{\text{study on beach}} = \mu_{\text{study underwater}} \)
2. When looking at the main effect of one IV, you should ignore the existence of the other IV
3. Compare all conditions that have one level of the IV to all conditions that have the other level of the IV

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Main Effects

1. For the main effect of where the words were studied:
   - Is the average of the values of the left two bars (conditions in which people studied on the beach), \( (15 + 10) / 2 = 12.5 \), different from the average of the values of the right two bars (conditions in which people studied underwater), \( (8 + 5) / 2 = 6.5 \)?
   - Remember \( \bar{X} = \mu \)

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Main Effects

1. For the main effect of where the words were recalled:
   - Is the average of the values of the green bars (conditions in which people recalled on the beach), \( (15 + 8) / 2 = 11.5 \), different from the average of the values of the blue bars (conditions in which people recalled underwater), \( (10 + 5) / 2 = 7.5 \)?
   - Remember \( \bar{X} = \mu \)
Interactions

The third question that we can ask about a factorial design is:
- Are the effects of the two IVs independent of each other?

This type of effect is called an interaction effect or just an interaction.
- An interaction between two factors occurs whenever the mean differences between individual treatment conditions, or cells, are different from what would be predicted from the overall main effects of the factors.

Definitions of Interaction

There are several equivalent ways of defining interaction:
- When the effect of one factor depends on the different levels of a second factor, then there is an interaction between the factors.
- When the results of a two-factor study are presented in a graph, the existence of nonparallel lines (line that cross or converge) indicates an interaction between the two factors.

Simple Main Effect Definition of Interaction

An interaction occurs when the nature of the simple main effect of one IV depend on then level of the other IV.
- Does the effect of whether you study dry or wet depend on whether you recall dry or wet?
Simple Main Effect Definition of *Interaction*

- Yes
- If you are recalling on the beach, then studying on the beach is better than studying underwater
- If you are recalling underwater, then studying on the beach is poorer than studying underwater

The simple main effect of whether you do better studying on the beach or underwater depends on whether your recall on the beach or underwater

Thus, the two variables (where you study and where you recall) interact

Graphical Definition of *Interaction*

When the lines or bars on a graph are not parallel, then an interaction has occurred
Factorial ANOVA

- The factorial analysis of variance answers each of the questions that can be asked
  - Is there a main effect of the first IV?
  - Is there a main effect of the second IV?
  - Is there an interaction effect of the two IVs?
- The ANOVA accomplishes these goals by giving us an F ratio for each of the questions that are asked

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Factorial ANOVA

- \( H_0 \) and \( H_1 \) for each of the main effects takes the same form as the single factor ANOVA \( H_0 \) and \( H_1 \):
  - \( H_0 \): \( \mu_1 = \mu_2 = \ldots = \mu_k \)
  - \( H_1 \): not \( H_0 \)
- \( H_0 \) and \( H_1 \) for the interaction take the following form:
  - \( H_0 \): there is no interaction
  - \( H_1 \): there is an interaction
- Specify \( \alpha = .05 \)

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Factorial ANOVA Example

- Participants learn a list of 20 words either while sitting on the beach or while snorkeling underwater
- Participants get a recognition test of the 20 words either while sitting on the beach or while snorkeling underwater
- There are five participants per condition and each participant participates in only one condition
Factorial ANOVA Example

- Main effect of where you study
  - \( H_0: \mu_{\text{study on beach}} = \mu_{\text{study underwater}} \)
  - \( H_1: \mu_{\text{study on beach}} \neq \mu_{\text{study underwater}} \)
- Main effect of where you recognize
  - \( H_0: \mu_{\text{recognize on beach}} = \mu_{\text{recognize underwater}} \)
  - \( H_1: \mu_{\text{recognize on beach}} \neq \mu_{\text{recognize underwater}} \)
- Interaction of where you study and recognize
  - \( H_0: \) There is no interaction
  - \( H_1: \) There is an interaction
- \( \alpha = .05 \)

Factorial ANOVA Summary Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between treatments</td>
<td>113.75</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study (factor 1)</td>
<td>1.25</td>
<td>1</td>
<td>1.25</td>
<td>0.5</td>
</tr>
<tr>
<td>Recognize (factor 2)</td>
<td>11.25</td>
<td>1</td>
<td>11.25</td>
<td>4.3</td>
</tr>
<tr>
<td>Study X Recognize (factor 1 X factor 2)</td>
<td>101.25</td>
<td>1</td>
<td>101.25</td>
<td>40.3</td>
</tr>
<tr>
<td>Within treatments</td>
<td>40</td>
<td>16</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>153.75</td>
<td>19</td>
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\( SS_{\text{between}} = n \cdot SS_{\text{M}} \)
- \( \Sigma M = 12 + 7 + 6 + 10 = 35 \)
- \( \Sigma M^2 = 12^2 + 7^2 + 6^2 + 10^2 = 292 \)
- \( SS_{\text{M}} = 329 - 35^2/4 = 22.75 \)
- \( SS_{\text{between}} = 5 \cdot 22.75 = 113.75 \)
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1. SS<sub>Study</sub> = n - SS<sub>Marginal Means for Study</sub>
2. SS<sub>Marginal Means for Study</sub> = 9 + 8.5 = 17.5
3. SS<sub>Recognize</sub> = n - SS<sub>Marginal Means for Recognize</sub>
4. SS<sub>Marginal Means for Recognize</sub> = 9.5 + 8 = 17.5

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4. SS<sub>Marginal Means for Recognize</sub> = 9.5 + 8 = 17.5
5. SS<sub>Study</sub> = 10 - 0.125 = 1.25

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1. SS<sub>Study X Recognize</sub> = SS<sub>Between</sub> - SS<sub>Study</sub> - SS<sub>Recognize</sub>
2. SS<sub>Between</sub> = 113.75 - 1.25 - 11.25 = 101.25
3. SS<sub>Total</sub> = ΣX<sup>2</sup><sub>Grand</sub> - (ΣX<sub>Grand</sub>)<sup>2</sup> / N
   = 1685 - 175<sup>2</sup> / 20
   = 153.75
4. SS<sub>Between</sub> + SS<sub>Within</sub> = 113.75 + 40 = 153.75
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- df_Between = # of conditions – 1 = 4 – 1 = 3
- df_Stud = # of levels of Study – 1 = 2 – 1 = 1
- df_Recog = # of levels of Recognize – 1 = 2 – 1 = 1
- df_Stud X Recog = df_Stud X df_Recog = 1 X 1 = 1
- df_Within = Σ(n_ – 1) = (5 – 1) + (5 – 1) + (5 – 1) + (5 – 1) = 16
- df_Total = # of scores – 1 = 20 – 1 = 19 = df_Between + df_Within + 3 + 16

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- MS_Stud = SS_Stud / df_Stud = 1.25 / 1 = 1.25
- MS_Recog = SS_Recog / df_Recog = 11.25 / 1 = 11.25
- MS_Stud X Recog = SS_Stud X Recog / df_Stud X Recog = 101.25 / 1 = 101.25
- MS_Within = SS_Within / df_Within = 40 / 16 = 2.5

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- F_Stud = MS_Stud / MS_Within = 1.25 / 2.5 = 0.5
- F_Recog = MS_Recog / MS_Within = 11.25 / 2.5 = 4.5
- F_Stud X Recog = MS_Stud X Recog / MS_Within = 101.25 / 2.5 = 40.5
Two Factor ANOVA Decisions

- Determine the critical value of $F$ from a table
  - Need, $df$ for the effect you want (factor 1, factor 2, factor 1 X factor 2), $df_{within\ treatments}$ and $\alpha$

For the main effect of where you study
  - $df_{study} = 1$, $df_{within\ treatments} = 16$, $\alpha = .05$, critical $F = 4.49$
  - If calculated / observed $F$ is at least as large as the critical $F$, then reject $H_0$
    - Calculated $F = 0.5$, critical $F = 4.49$, fail to reject $H_0$
    - There is insufficient evidence to suggest that where you study (on the beach or under water) influences the number of words recognized

For the main effect of where you recognize
  - $df_{recognize} = 1$, $df_{within\ treatments} = 16$, $\alpha = .05$, critical $F = 4.49$
  - If calculated / observed $F$ is at least as large as the critical $F$, then reject $H_0$
    - Calculated $F = 4.5$, critical $F = 4.49$, reject $H_0$
    - There is sufficient evidence to suggest that where you recognize the words (on the beach or under water) influences the number of words recognized
Two Factor ANOVA Decisions

- For the interaction of where you study and where you recognize
  \[ \text{df}_{\text{study \times recognize}} = 1, \text{ df}_{\text{within treatments}} = 16, \alpha = .05, \]  
  \[ \text{critical } F = 4.49 \]

- If calculated / observed F is at least as large as the critical F, then reject \( H_0 \)
  \[ \text{Calculated } F = 40.5, \text{ critical } F = 4.49, \text{ reject } H_0 \]

There is sufficient evidence to suggest that where you study the words (beach vs. underwater) and where you recognize the words (beach vs. underwater) interact.

ANOVA Assumptions

- Observations within each sample must be independent
- Population from which samples are selected must be normally distributed
- Populations from which the samples are selected must have equal variances

Higher-Order Factorial Designs

- ANOVA is not limited to just 2 IVs -- you can have as many IVs as is necessary to answer your questions
  - Because of the complexity of the design, you will rarely see more than 3 or 4 IVs used in a single experiment
- When there are more than two IVs, you get information on main effects, simple interactions and higher-order interactions
Higher Order Interactions

- A higher-order interaction occurs when the nature of the lower order interaction (such as the two variable interaction we have talked about) depends on the level of a third (or higher) IV.
- Higher order interactions are often difficult to interpret.

No Higher Order Interaction

![Graph showing no higher order interaction for Science Majors and Non-Science Majors.]

Higher Order Interaction

![Graph showing higher order interaction for Science Majors and Non-Science Majors.]