4. Let \( f \) be a function that is continuous on \([-1, 1]\) and differentiable on \((-1, 1)\). Show that if \( f'(x) > 0 \) for all \( x \in (-1, 1) \) then \( f(-1) < f(1) \).

By the Mean Value Theorem, there is \( c \in (-1, 1) \) such that

\[
\frac{f(1) - f(-1)}{1 - (-1)} = f'(c)
\]

so \( f(1) > f(-1) \).

5. Let \( R \) be a rectangle in the first quadrant that has one side on the \( y \)-axis, one side on the \( x \)-axis, and one vertex on the graph of \( y = 9 - x^2 \). Find the largest possible area of \( R \), and the dimensions of \( R \) which result in this area.

Let \( x \) be the length of the side along the \( x \)-axis, and let \( y \) be the length of the side along the \( y \)-axis. \( 0 \leq x \leq 3 \)

The area of \( R \) is \( A = xy = x(9-x^2) \) (since \((x,y)\) is on the graph)

\[
A' = 9 - 3x^2 = 3(3-x^2) \quad A' = 0 \text{ when } x = \pm \sqrt{3}
\]

\[
A(0) = 0 \quad A(\sqrt{3}) = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}
\]

The maximum value of \( f \) is \( 6\sqrt{3} \). It occurs when \( x = \sqrt{3} \) and \( y = 9 - 3 = 6 \).